Chapter 13

Measuring the Properties of Stars

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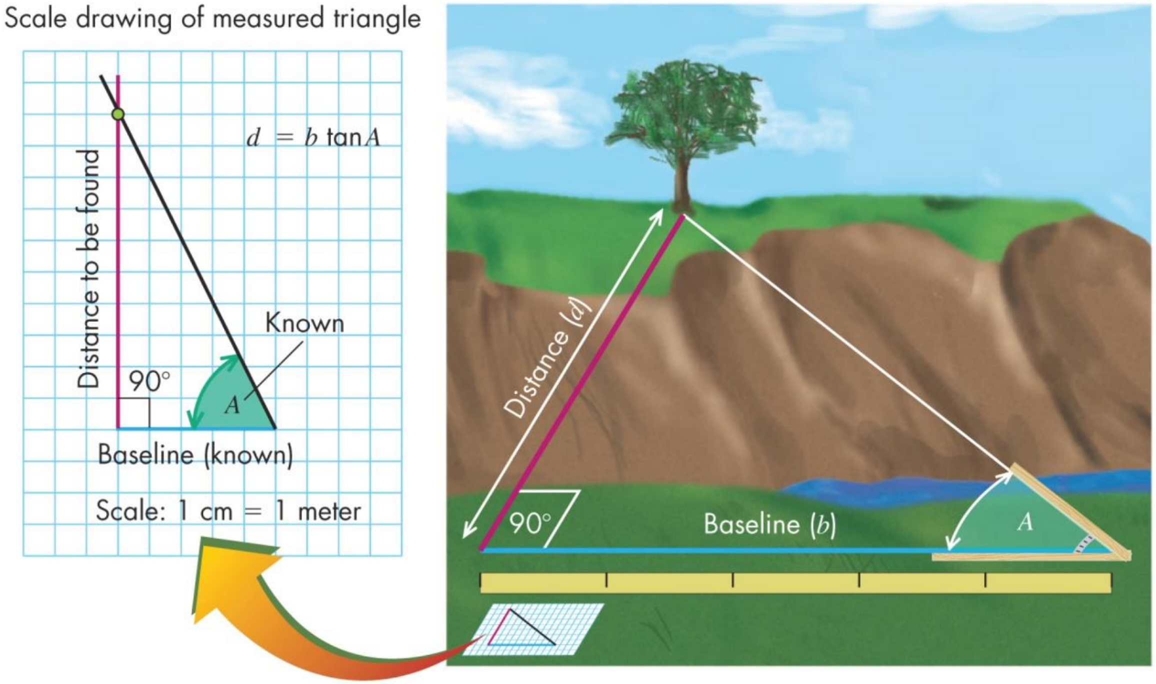
The Family of Stars

- Those tiny glints of light in the night sky are in reality huge, dazzling balls of gas, many of which are vastly larger and brighter than the Sun.
- They look dim because of their vast distances.
- Astronomers cannot probe stars directly, and consequently must devise indirect methods to ascertain their intrinsic properties.
- Measuring distances to stars and galaxies is not easy.
- Distance is <u>very</u> important for determining the intrinsic properties of astronomical objects.

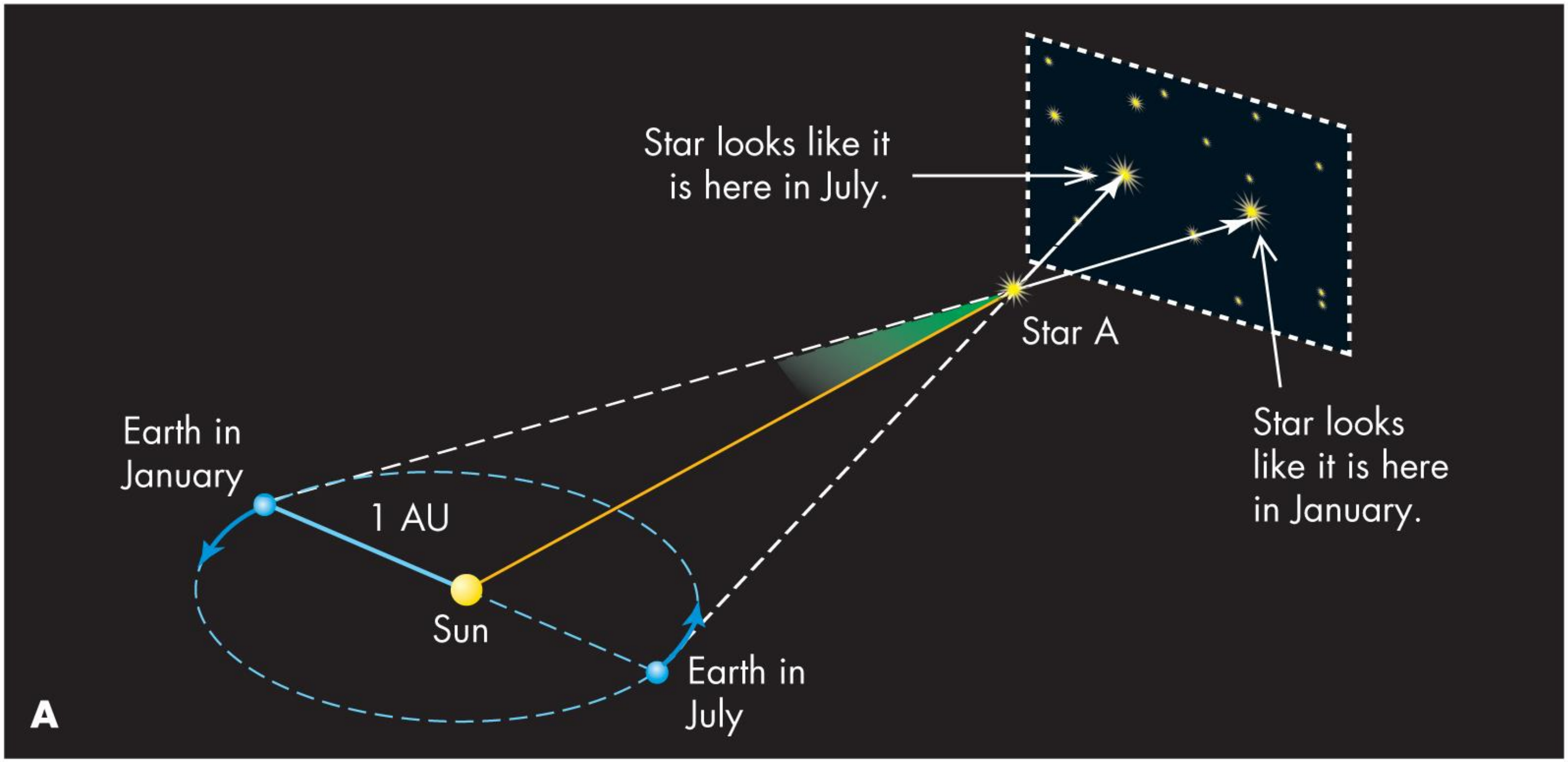
Triangulation



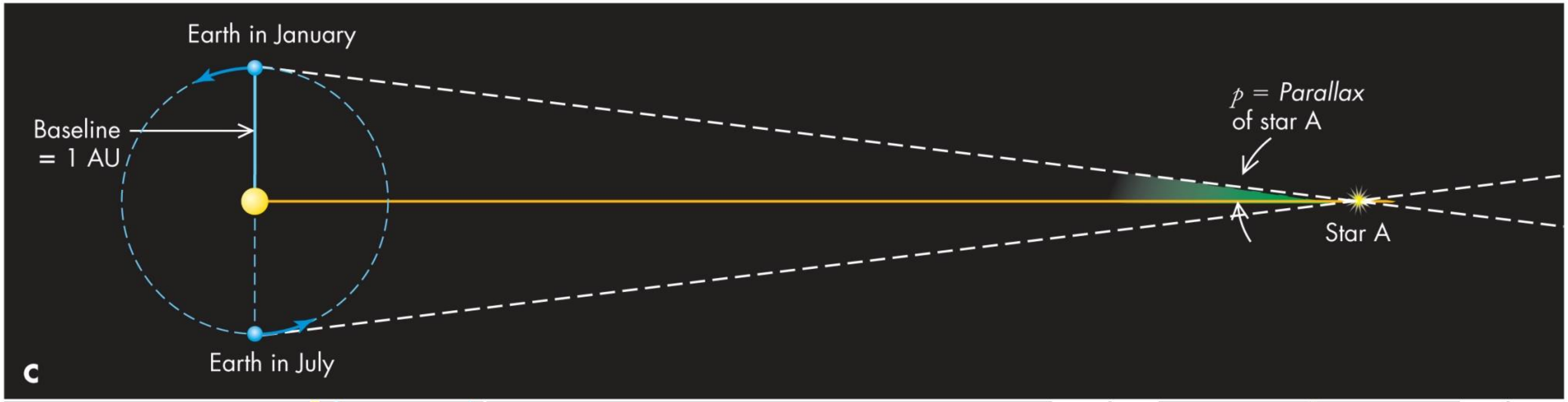
Measuring Distance by Triangulation and Parallax

- Fundamental method for measuring distances to nearby stars is ***triangulation:***
  - Measure length of a triangle's "baseline" and the angles from the ends of this baseline to a distant object
  - Use trigonometry or a scaled drawing to determine distance to object

Trigonometric Parallax

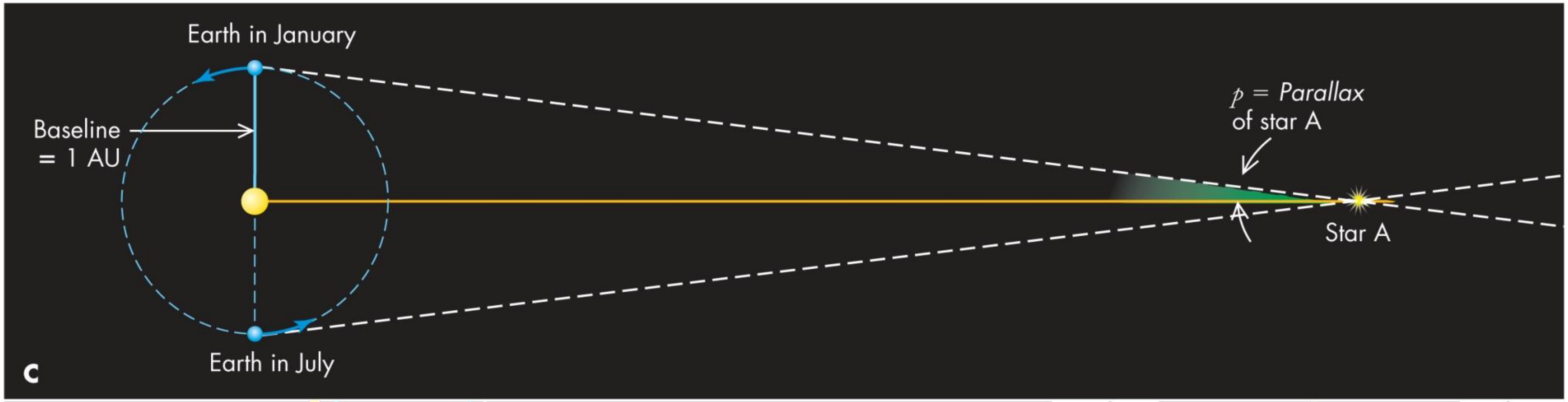


Calculating Distance Using Parallax



- A method of triangulation used by astronomers is called ***parallax:***
  - Baseline is the Earth's orbit radius (1 AU)
  - Angles measured with respect to very distant stars

The parsec (parallax-second)



- The shift of nearby stars is small, so angles are measured in arc seconds
- The parallax angle, *p*, is half the angular shift of the nearby star, and its distance in parsecs is given by:

$$d_{pc} = 1/p_{arc\ seconds}$$

- A ***parsec*** is 3.26 light-years (3.09×10^{13} km)
- Useful only to distances of about 250 parsecs

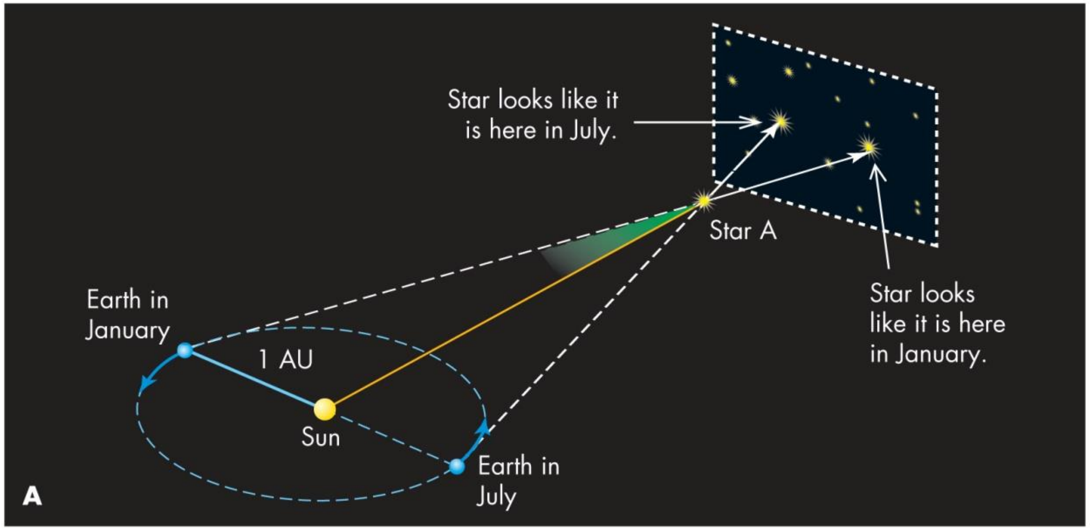
Example: Distance to Sirius

- Measured parallax angle for Sirius is 0.377 arc second
- From the formula,

d_{pc} = 1/0.377

= 2.65 parsecs

= 8.6 light-years



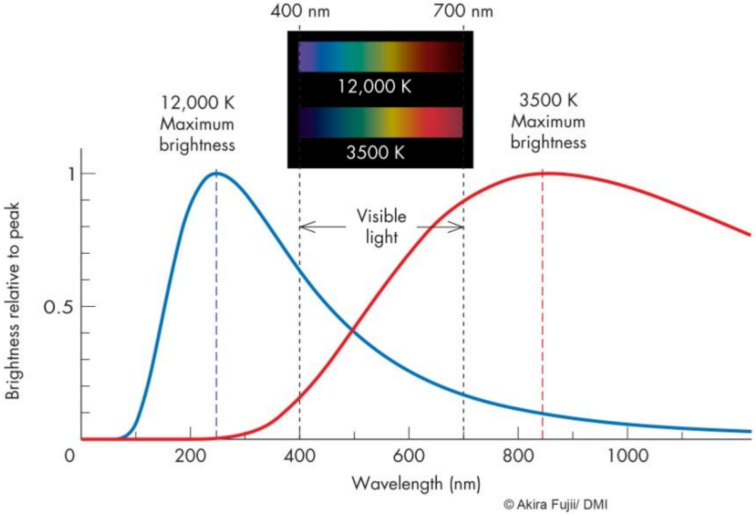
Light, the Astronomer's Tool

- Astronomers want to know the motions, sizes, colors, and structures of stars.
- This information helps to understand the nature of stars as well as their life cycle.
- The light from stars received at Earth is all that is available for this analysis.

(a): NASA, ESA, and the Hubble Heritage Team (STScI/AURA)

Courtesy of Dean Ketelsen

Temperature



- The color of a star indicates its relative temperature – blue stars are hotter than red stars.
- More precisely, a star's surface temperature (in Kelvin) is given by the wavelength in nanometers (nm) at which the star radiates most strongly.

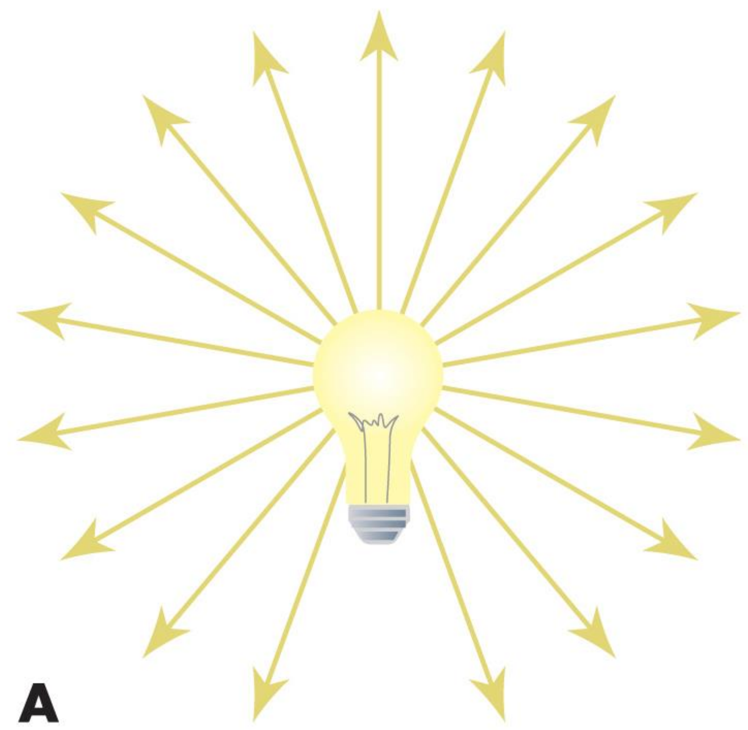
Key Measurement: Luminosity

(a): NASA, ESA, and the Hubble Heritage Team (STScI/AURA)

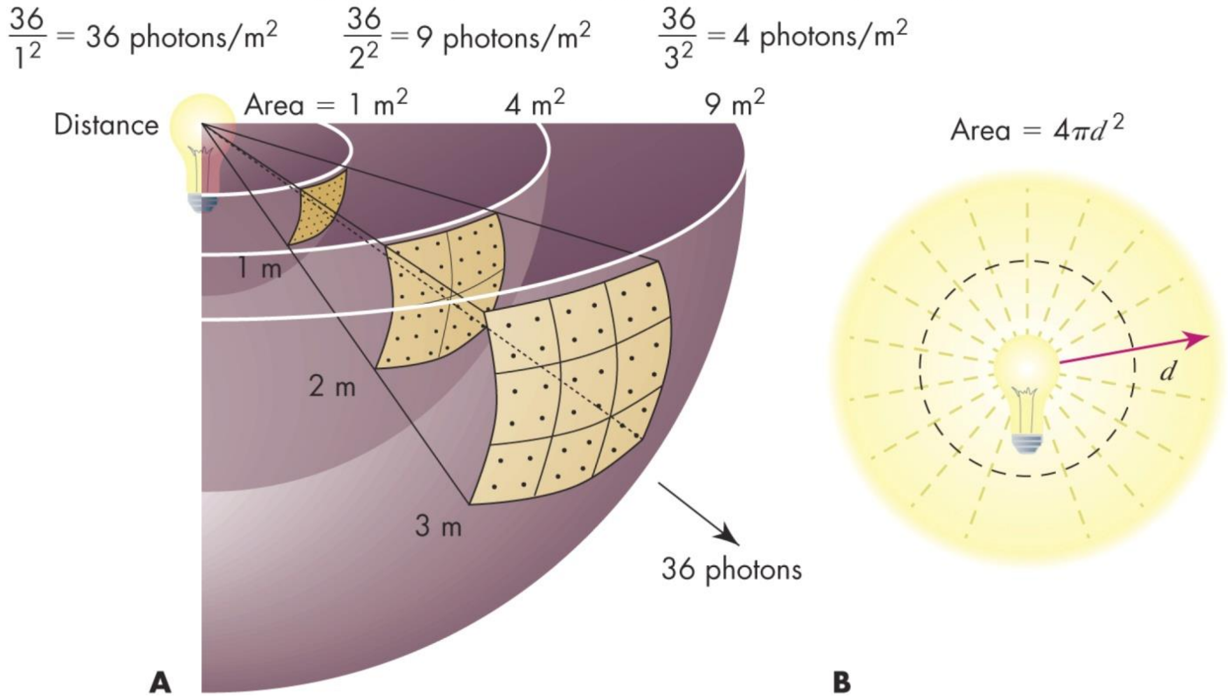
- Luminosity is a measure of a star's energy production (or hydrogen fuel consumption)
- Knowing a star's luminosity will allow a determination of a star's distance and radius

Luminosity

- The amount of energy a star emits each second is its ***luminosity*** (usually abbreviated as *L*).
- A typical unit of measurement for luminosity is the watt.
- Compare a 100-watt bulb to the Sun's luminosity, 4×10^{26} watts



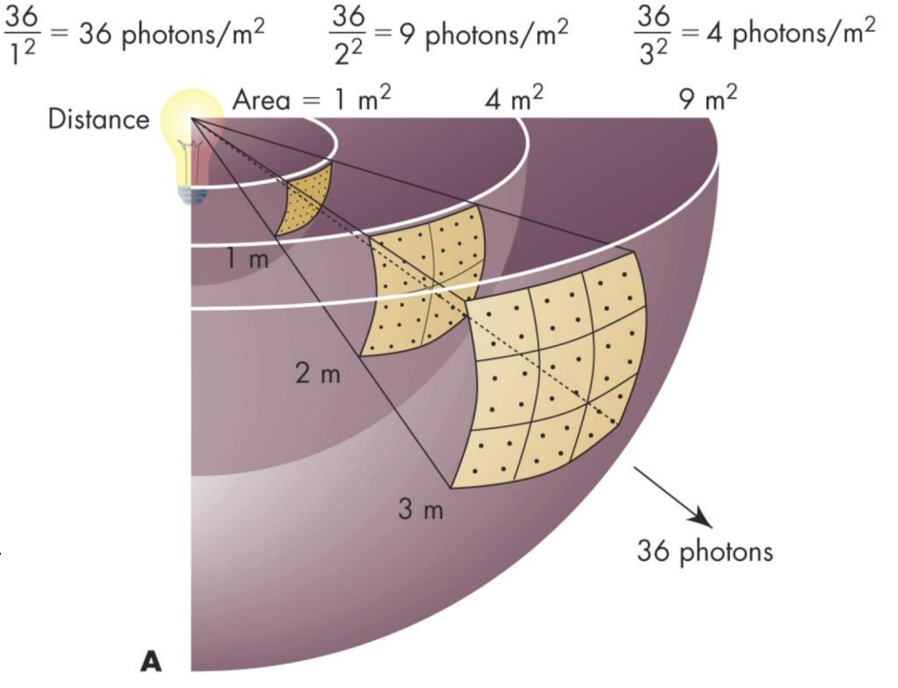
The Inverse-Square Law



- The ***inverse-square law*** relates an object's luminosity to its distance and its apparent brightness (how bright it appears to us)

Why distant objects appear dim!

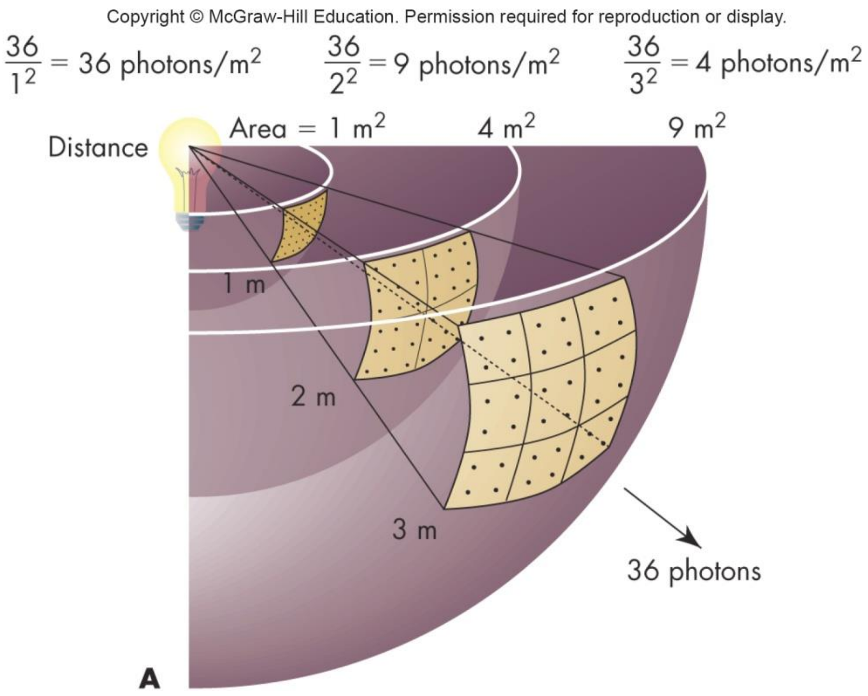
- This law can be thought of as the result of a fixed number of photons, spreading out evenly in all directions as they leave the source
- The photons have to cross larger and larger concentric spherical shells.
- For a given shell, the number of photons crossing it decreases per unit area



Brightness

- The inverse-square law (IS) is:

$$B = \frac{L}{4\pi d^2}$$



- *B* is the brightness at a distance *d* from a source of luminosity *L*
- This relationship is called the inverse-square law because the distance appears in the denominator as a square

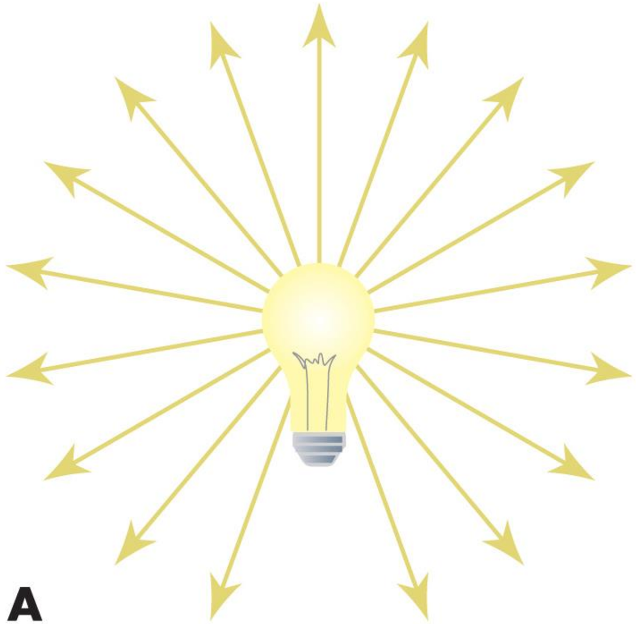
Inferring Luminosity

- The inverse-square law is one of the most important mathematical tools available to astronomers:
  - Given d from parallax measurements, a star's *L* can be found (A star's *B* can easily be measured by an electronic device, called a photometer, connected to a telescope.)
  - Or if *L* is known in advance, a star's distance can be found

(a): NASA, ESA, and the Hubble Heritage Team (STScI/AURA)

$$B = \frac{L}{4\pi d^2}$$

The "Standard Candle" Method



- If an object's ***intrinsic*** brightness is known, its distance can be determined from its ***observed*** brightness.
- Astronomers call this method of distance determination the ***method of standard candles.***
- This method is the principle manner in which astronomers determine distances in the universe.

Method of Standard Candles

- Step 1: Measure a star’s brightness (B) with a photometer
- Step 2: Determine star’s Luminosity, L
  - Some kinds of stars (variable stars, for example) or supernovae (Type 1a supernova) have a known luminosity.
- Use combined formula to calculate d, the distance to the star

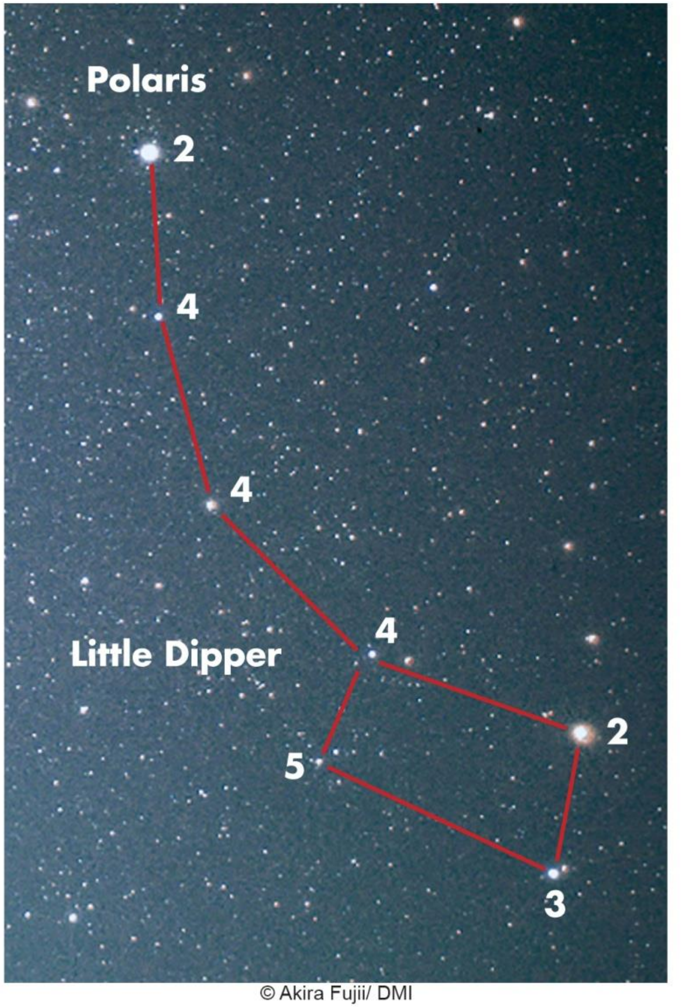
$$d = \sqrt{\frac{L}{4\pi B}}$$

The Magnitude Scale

- About 150 B.C., the Greek astronomer Hipparchus measured apparent brightness of stars using units called ***magnitudes.***
  - Brightest stars had magnitude 1 and dimmest had magnitude 6.
  - The system is still used today and units of measurement are called apparent magnitudes to emphasize how bright a star looks to an observer.
- A star's apparent magnitude depends on the star's luminosity and distance – a star may appear dim because it is very far away or it does not emit much energy.

Magnitude differences

- The apparent magnitude can be confusing.
  - Scale runs “backward”: high magnitude = low brightness
  - Modern calibrations of the scale create negative magnitudes.
  - Magnitude differences equate to brightness ratios:
    - A difference of 5 magnitudes = a brightness ratio of 100
    - 1 magnitude difference = brightness ratio of $100^{1/5}$=2.512



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Absolute Magnitude

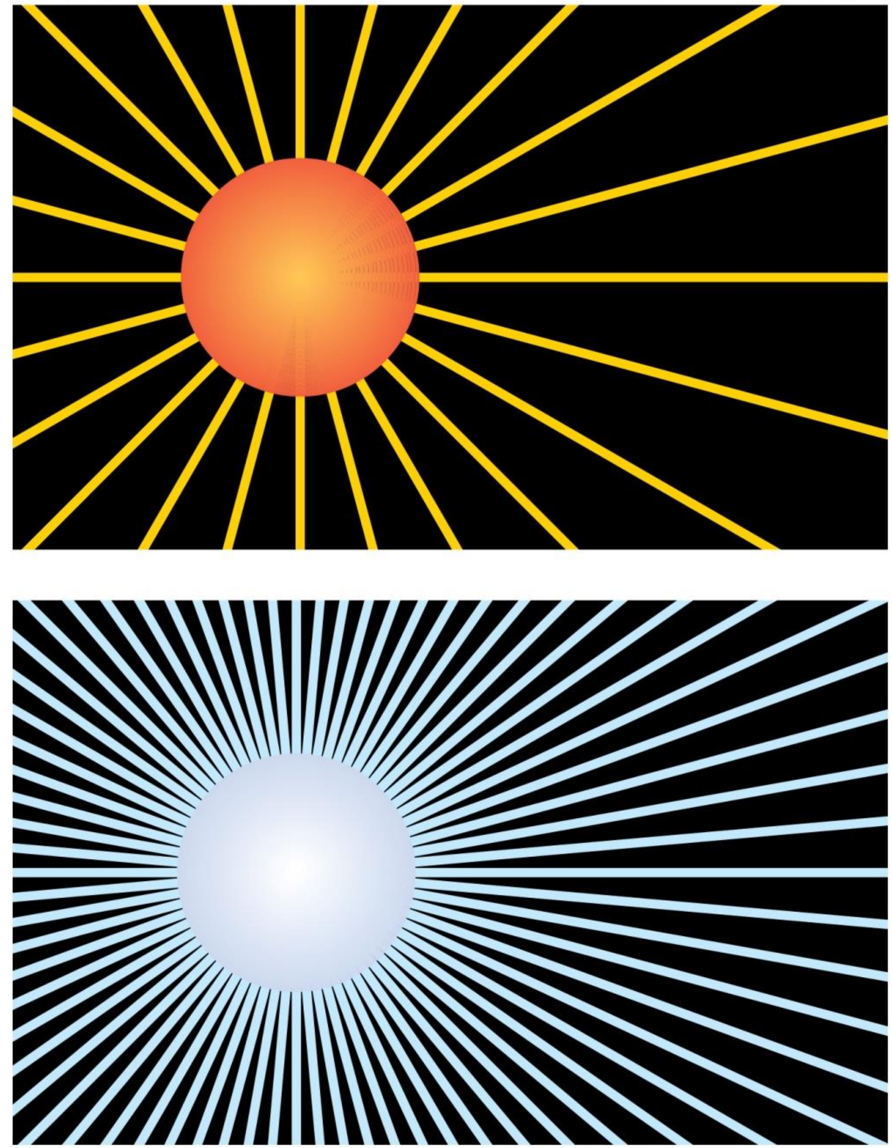
- Astronomers use ***absolute magnitude*** to measure a star's luminosity.
  - The absolute magnitude of a star is the apparent magnitude that same star would have at 10 parsecs.
  - A comparison of absolute magnitudes is now a comparison of luminosities, no distance dependence.
  - An absolute magnitude of 0 approximately equates to a luminosity of $100L_{\odot}$.

Stefan-Boltzmann Law

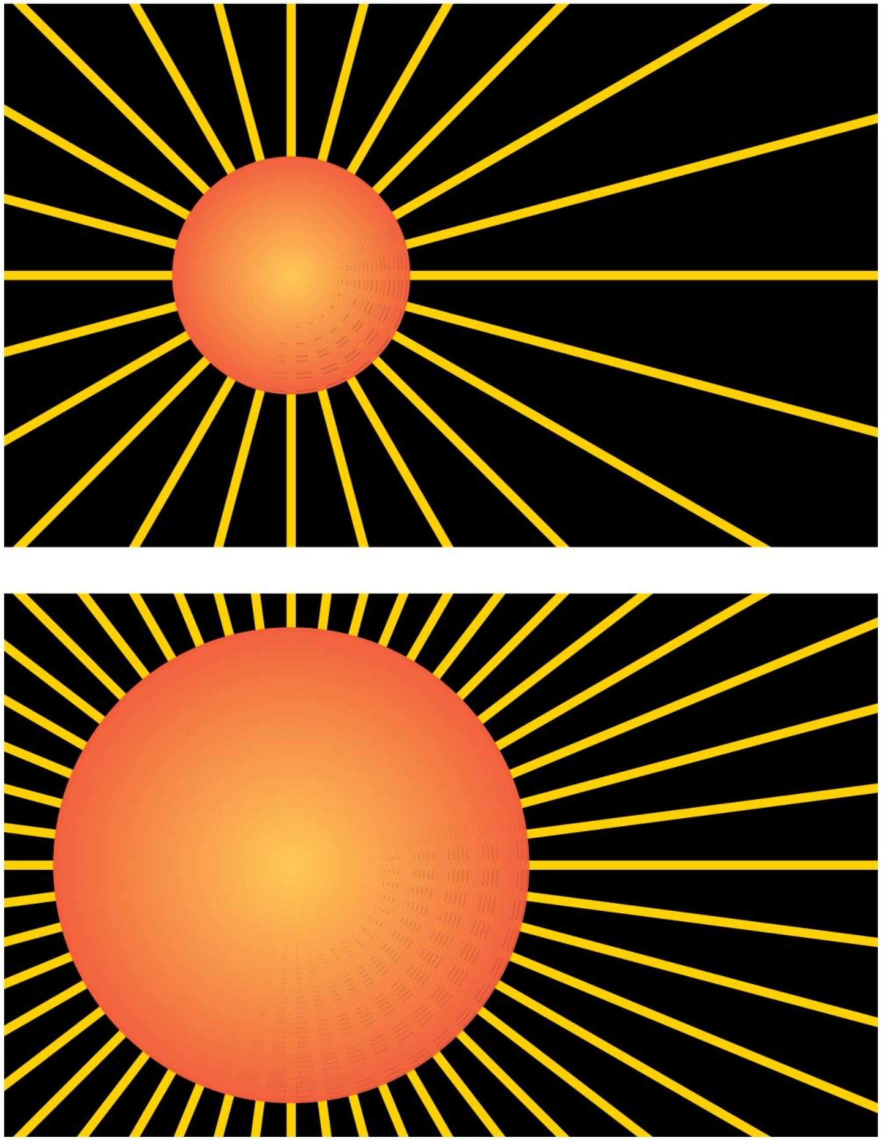
- If two stars have the same size but the surface of one is hotter than the other, the hotter star will have a greater luminosity.
- The Stefan-Boltzmann (SB) Law gives this:

$$B = \sigma T^4$$

- Here σ is the Stefan-Boltzmann constant (5.67 × 10^{-8} watts $m^{-2}K^{-4}$)



Radius



- If two stars have the same temperature but one is more luminous than the other, the more-luminous star must have a larger surface area, therefore a larger radius than the dimmer star.

Tying It All Together

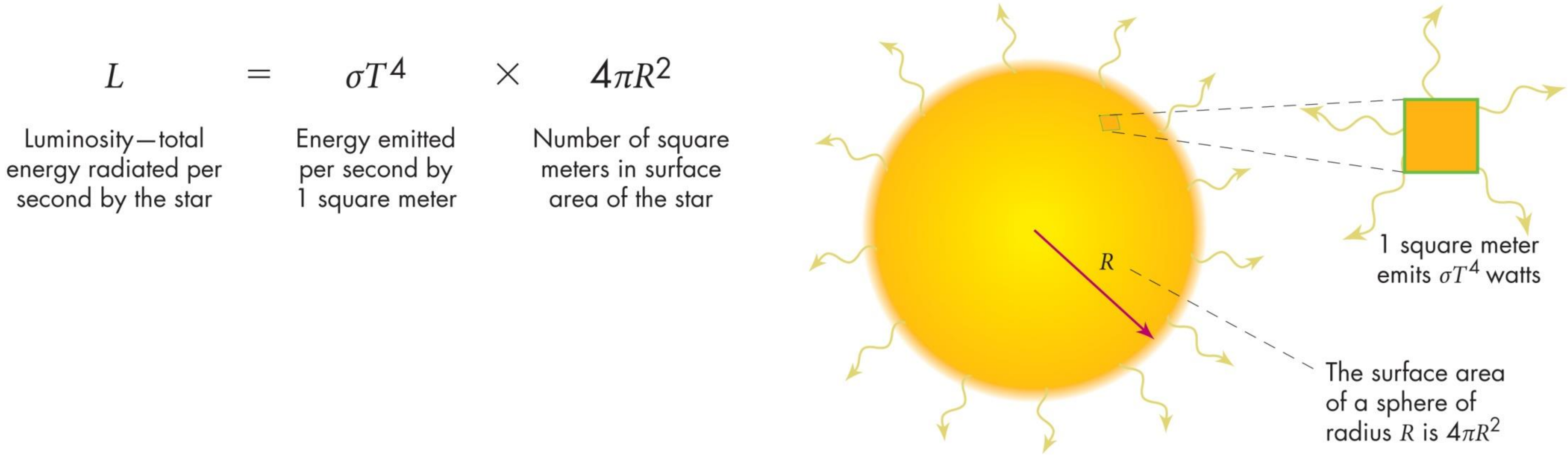
- The Stefan-Boltzmann law only applies to stars, but not hot, low-density gases.
- We can combine SB and IS to get:

$$L = 4\pi R^2 \sigma T^4$$

- *R* is the radius of the star.
- Given *L* and *T*, we can then find a star's radius!

(a): NASA, ESA, and the Hubble Heritage Team (STScI/AURA)

Finding Luminosity



Large Range in Stellar Radii

- The methods using the Stefan-Boltzmann law and interferometer observations show that stars differ enormously in radius.
  - Some stars are hundreds of times larger than the Sun and are referred to as ***giants.***
  - Stars smaller than the giants are called ***dwarfs.***

(a): NASA, ESA, and the Hubble Heritage Team (STScI/AURA)

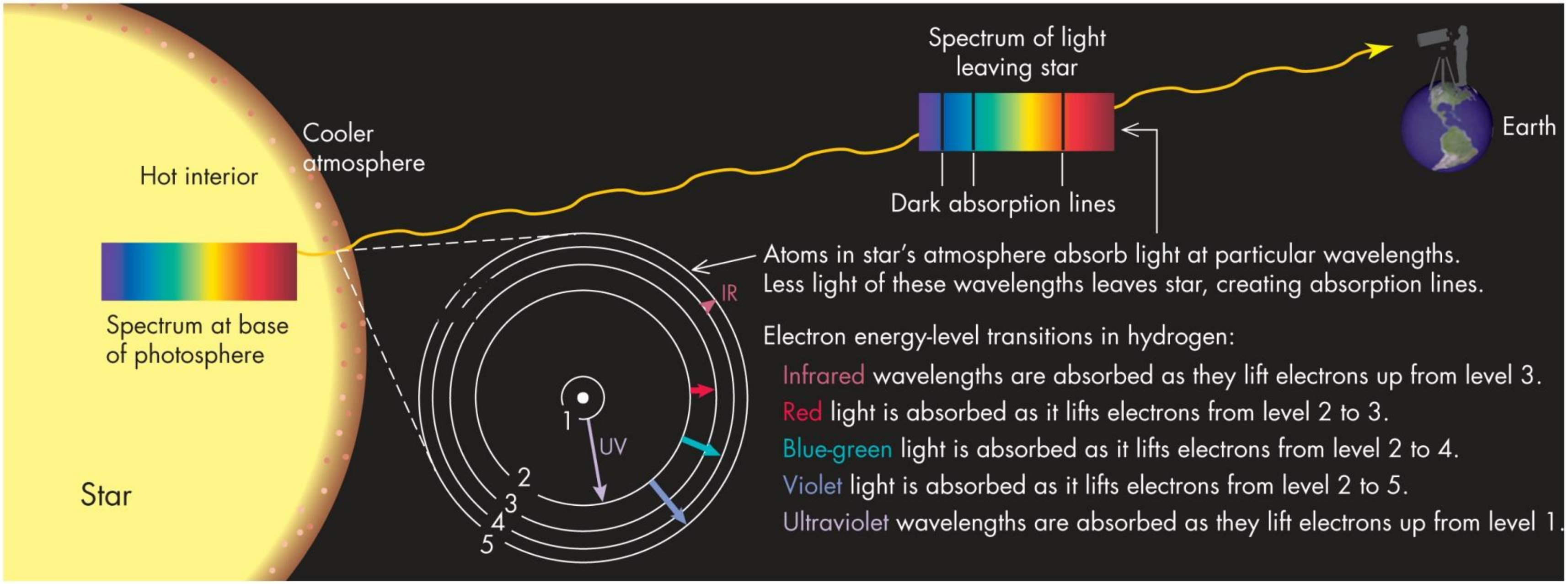
The Spectra of Stars

- A star’s spectrum typically depicts the energy it emits at each wavelength.
- A spectrum also can reveal a star’s composition, temperature, luminosity, velocity in space, rotation speed, and other properties.
- On certain occasions, it may reveal mass and radius.

Measuring a Star's Composition

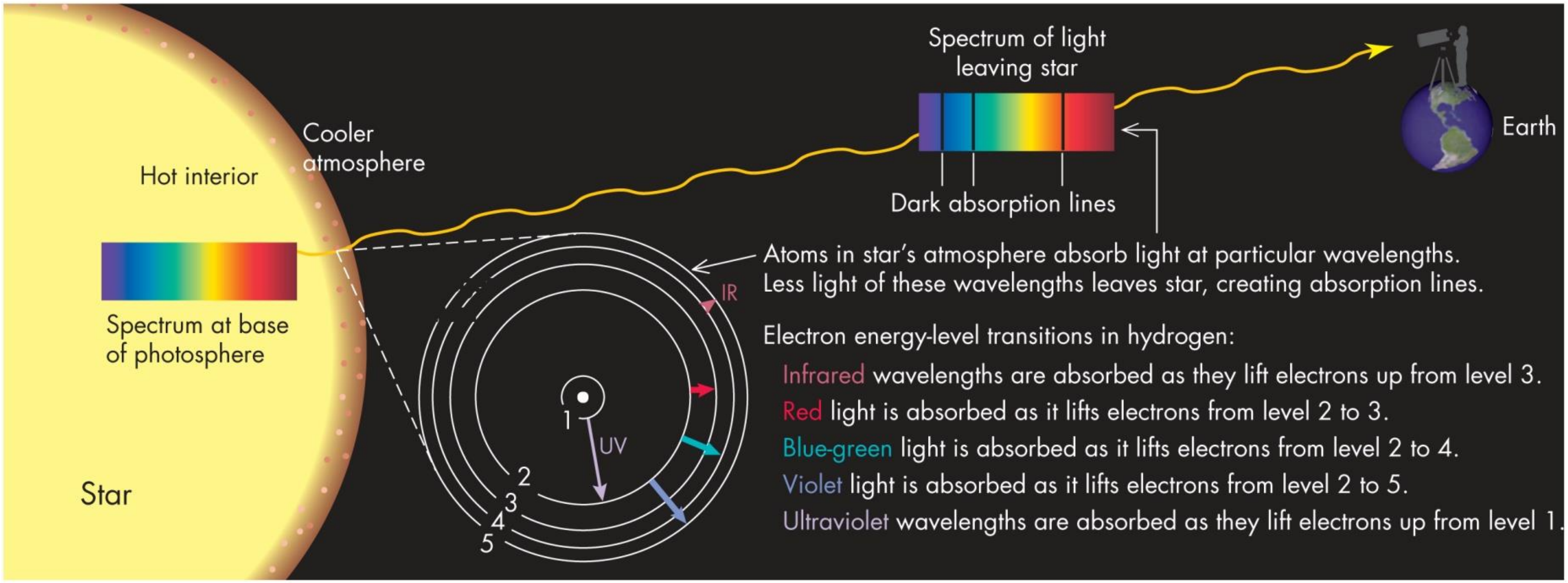
- As light moves through the gas of a star's surface layers, atoms absorb radiation at some wavelengths, creating dark absorption lines in the star's spectrum.
- Every atom creates its own unique set of absorption lines.
- Determining a star's surface composition is then a matter of matching a star's absorption lines to those known for atoms.

Strength of Absorption Line



- To find the quantity of a given atom in the star, we use the darkness of the absorption line
- This technique of determining composition and abundance can be tricky!

Temperature's Effect



- Possible overlap of absorption lines from several varieties of atoms being present
- Temperature can also affect how strong (dark) an absorption line is.

Temperature's Effect on Spectra

- A photon is absorbed when its energy matches the difference between two electron energy levels and an electron occupies the lower energy level.
- Higher temperatures, through collisions and energy exchange, will force electrons, on average, to occupy higher electron levels – lower temperatures, lower electron levels.

Temperature Can Erase Lines

- Consequently, absorption lines will be present or absent depending on the presence or absence of an electron at the right energy level and this is very much dependent on temperature.
- Adjusting for temperature, a star's composition can be found – interestingly, virtually all stars have compositions very similar to the Sun's: 71% H, 27% He, and a 2% mix of the remaining elements.

Early Classification of Stars

- Historically, stars were first classified into four groups according to their color (white, yellow, red, and deep red), which were subsequently subdivided into classes using the letters A through N.

Modern Classification of Stars

- Cecilia Payne then demonstrated the physical connection between temperature and the resulting absorption lines.

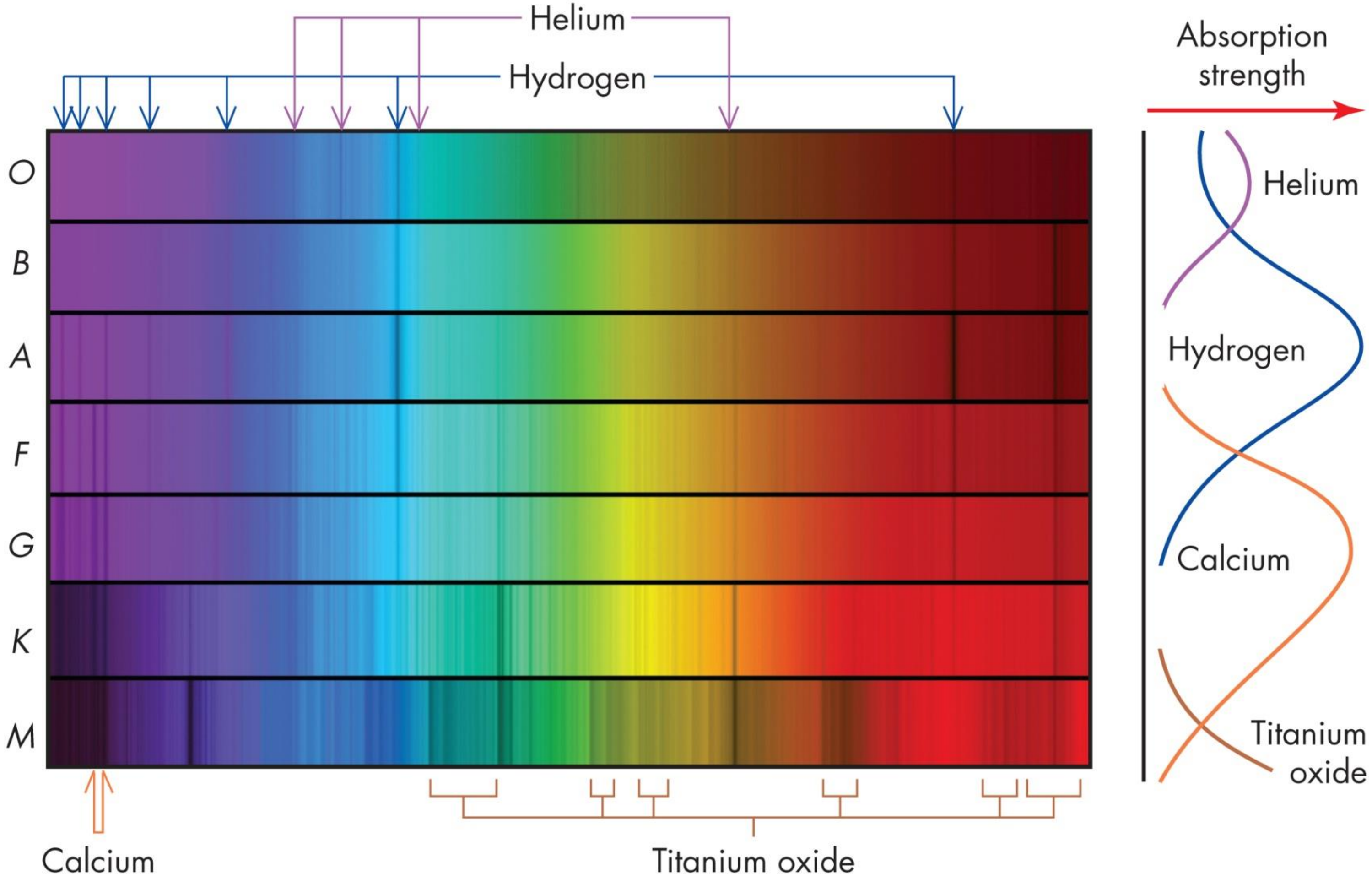
Courtesy of Katherine Haramundanis

Spectral Types

- Annie Jump Cannon discovered the classes were more orderly in appearance if rearranged by temperature – Her reordered sequence became O, B, A, F, G, K, M (O being the hottest and M the coolest) and are today known as ***spectral types.***

Courtesy of Harvard College Observatory

Absorption Lines Changing with Temperature



Spectral Classification

- O stars are very hot and the weak hydrogen absorption lines indicate that hydrogen is in a highly ionized state.
- A stars have just the right temperature to put electrons into hydrogen's 2^{nd} energy level, which results in strong absorption lines in the visible.
- F, G, and K stars are of a low enough temperature to show absorption lines of metals such as calcium and iron, elements that are typically ionized in hotter stars.
- K and M stars are cool enough to form molecules and their absorption "bands" become evident.

New Spectral Types

- Temperature range: more than 25,000 K for O (blue) stars and less than 3500 K for M (red) stars
- New spectral types L, T and Y have been added, even cooler than M stars
- Spectral classes subdivided with numbers - the Sun is G2

Measuring a Star's Motion

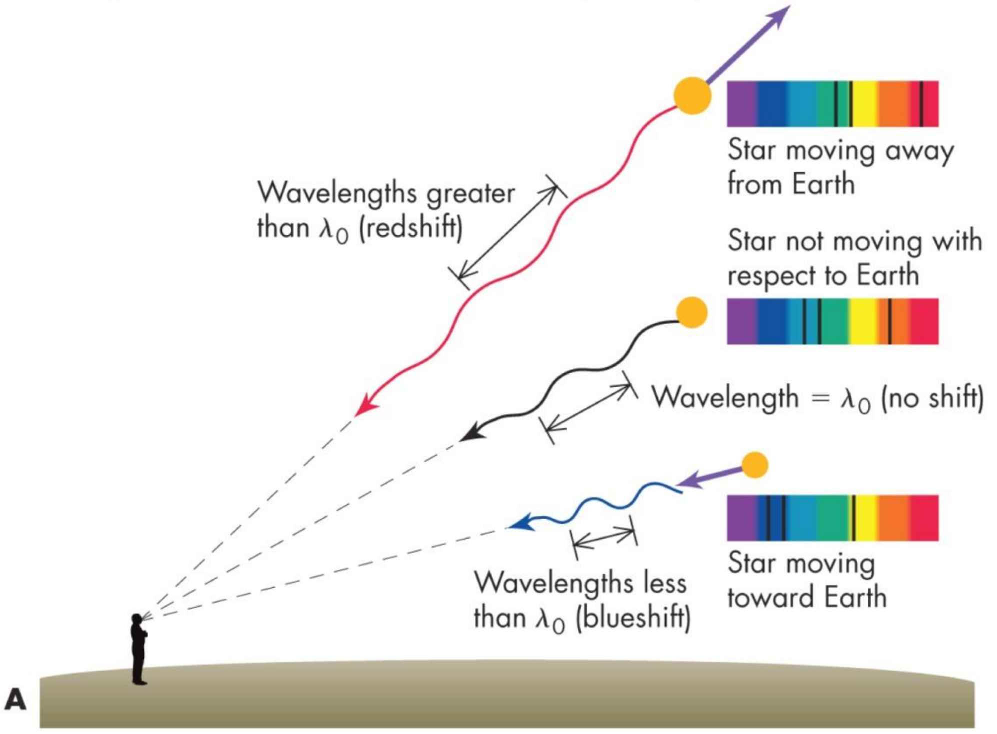
- A star's motion is determined from the Doppler shift of its spectral lines.
  - The amount of shift depends on the star's ***radial velocity*,** which is the star's speed along the line of sight.
  - Given that we measure $\Delta\lambda$, the shift in wavelength of an absorption line of wavelength λ, the radial speed v is given by:

$$v = \left(\frac{\Delta\lambda}{\lambda}\right)\mathrm{c}$$

  - c is the speed of light

Radial Motion

- Note that λ is the wavelength of the absorption line for an object at rest and its value is determined from laboratory measurements on nonmoving sources.
- An increase in wavelength means the star is moving away, a decrease means it is approaching – speed across the line on site cannot be determined from Doppler shifts.



Summary of Stellar Motion

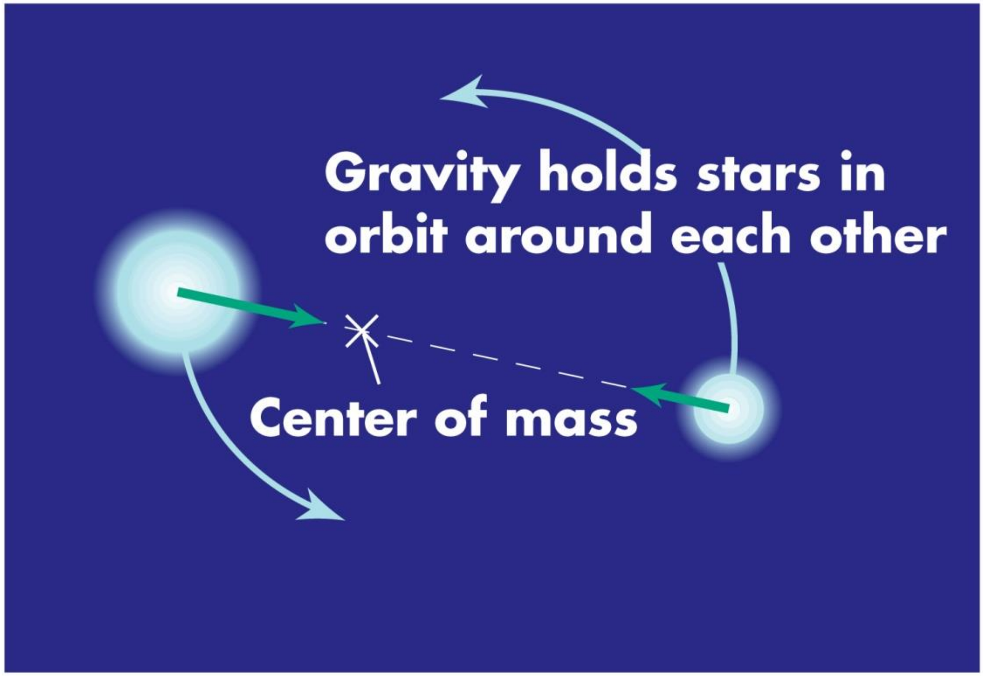
- Doppler measurements and related analysis show:
  - All stars are moving and that those near the Sun share approximately the same direction and speed of revolution (about 200 km/sec) around the center of our galaxy.
  - Superimposed on this orbital motion are small random motions of about 20 km/sec.
  - In addition to their motion through space, stars spin on their axes and this spin can be measured using the Doppler shift technique – young stars are found to rotate faster than old stars.
- Stars moving across our line of sight have no Doppler shift, although they do display a shift in position that astronomers call *proper motion*.

Binary Stars

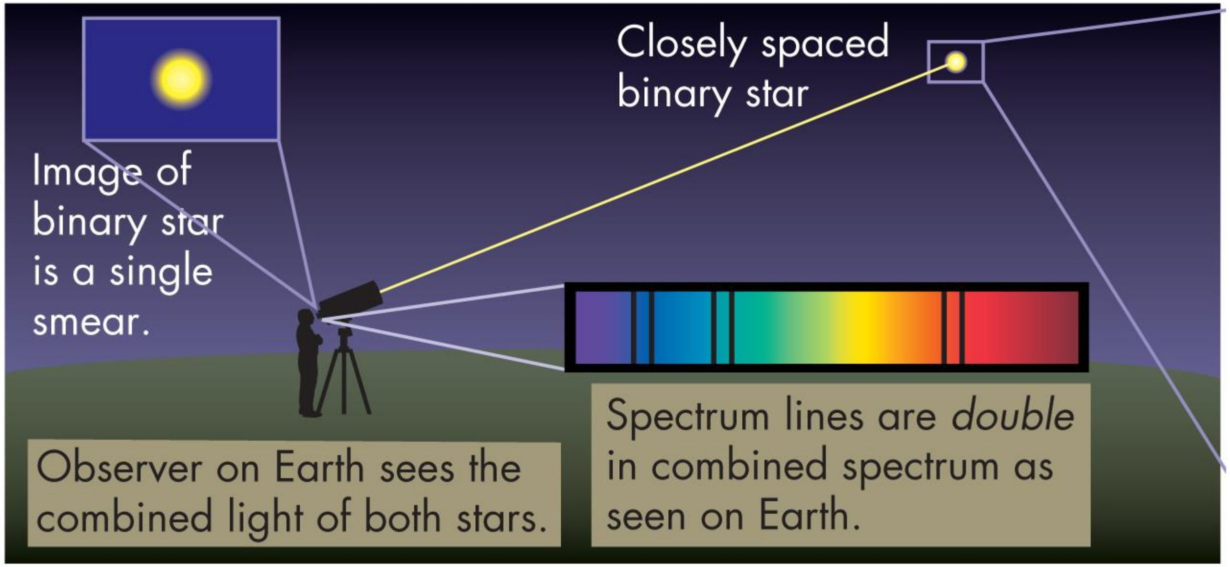
- Two stars that revolve around each other as a result of their mutual gravitational attraction are called ***binary stars.***
- Binary star systems offer one of the few ways to measure stellar masses – and stellar mass plays the leading role in a star's evolution.
- Of O and B type stars, 80% have orbiting companions (some more than one).
- Of Sun-like stars, roughly half are in binary systems.
- Most binary stars are only a few AU apart – a few are even close enough to touch.

Visual Binary Stars

- ***Visual binaries*** are binary systems where we can directly see the orbital motion of the stars about each other by comparing images made several years apart.



Spectroscopic Binaries



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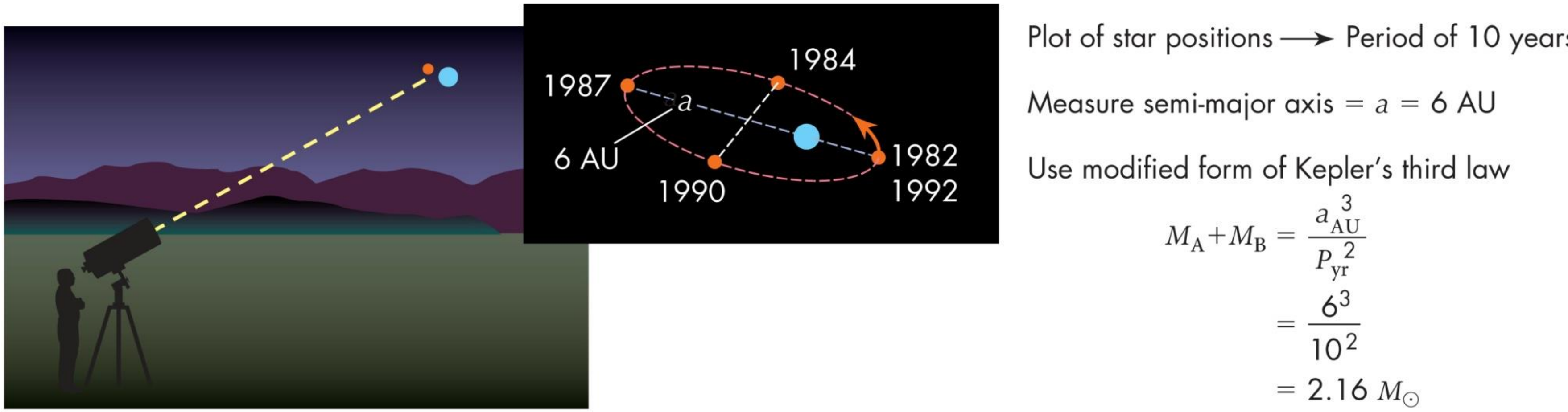
- ***Spectroscopic binaries*** are systems that are inferred to be binary by a comparison of the system's spectra over time
- Doppler analysis of the spectra can give a star's speed and by observing a full cycle of the motion the orbital period and distance can be determined

Newton's Form of Kepler's 3rd Law

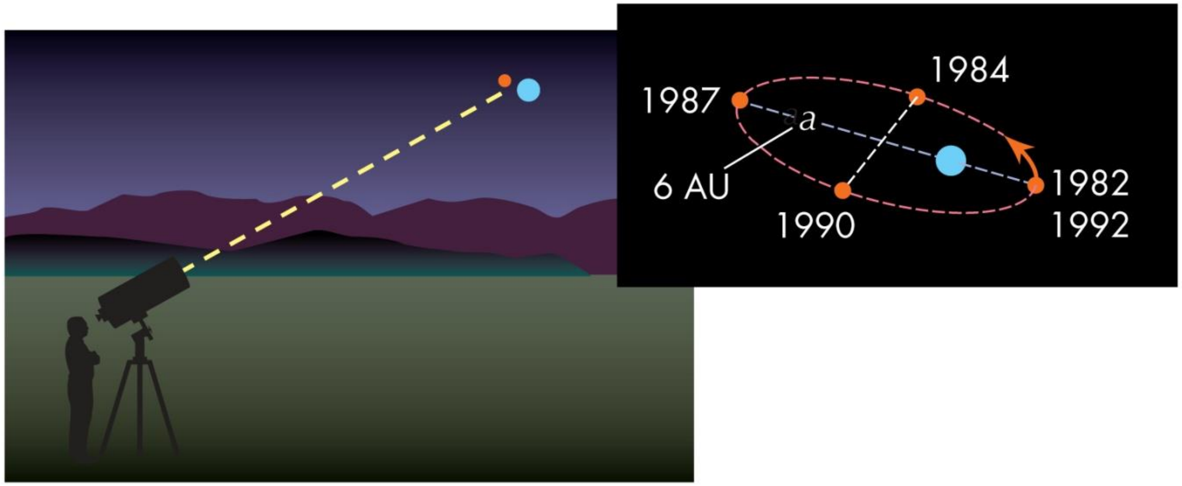


- Kepler's third law as modified by Newton is

$$M + m = \frac{a^3}{P^2}$$

- m and M are the binary star masses (in solar masses), P is their period of revolution (in years), and a is the semimajor axis of one star's orbit about the other (in AU)

Stellar Masses



Plot of star positions ⟶ Period of 10 years

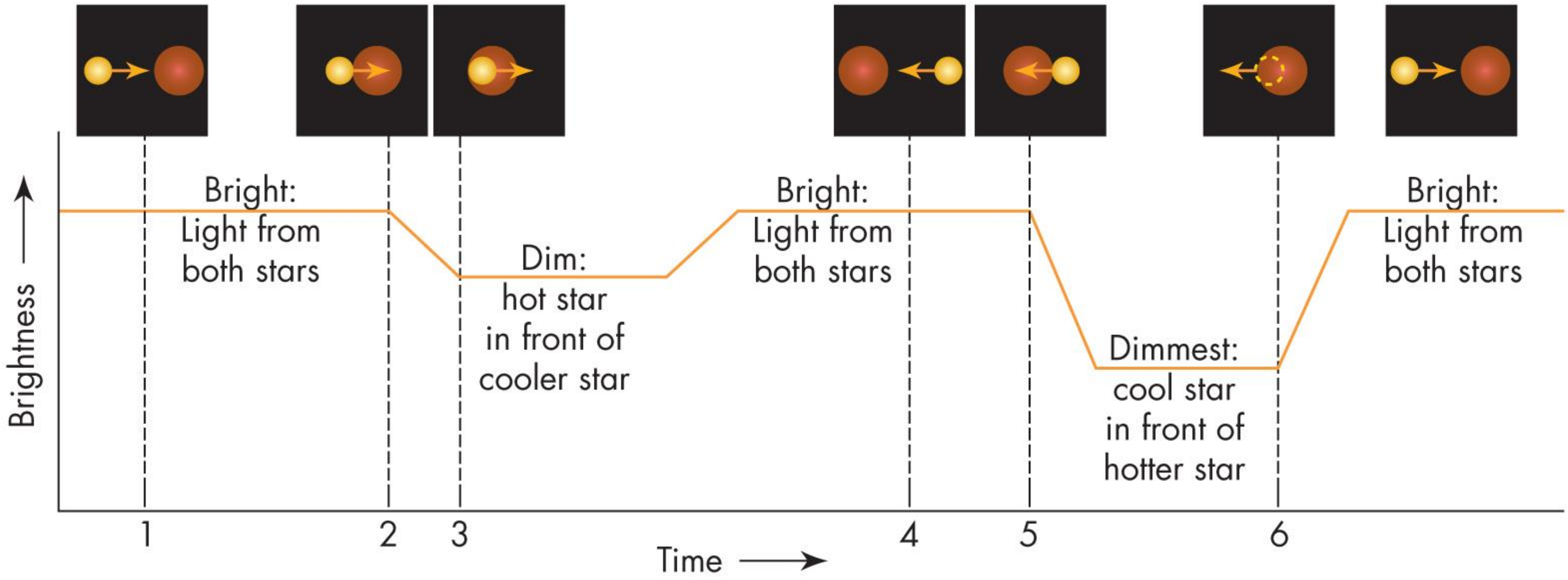
Measure semi-major axis = a = 6 AU

Use modified form of Kepler's third law

$$M_A + M_B = \frac{a_{AU}^{3}}{P_{yr}^{2}} = \frac{6^3}{10^2} = 2.16\ M_\odot$$

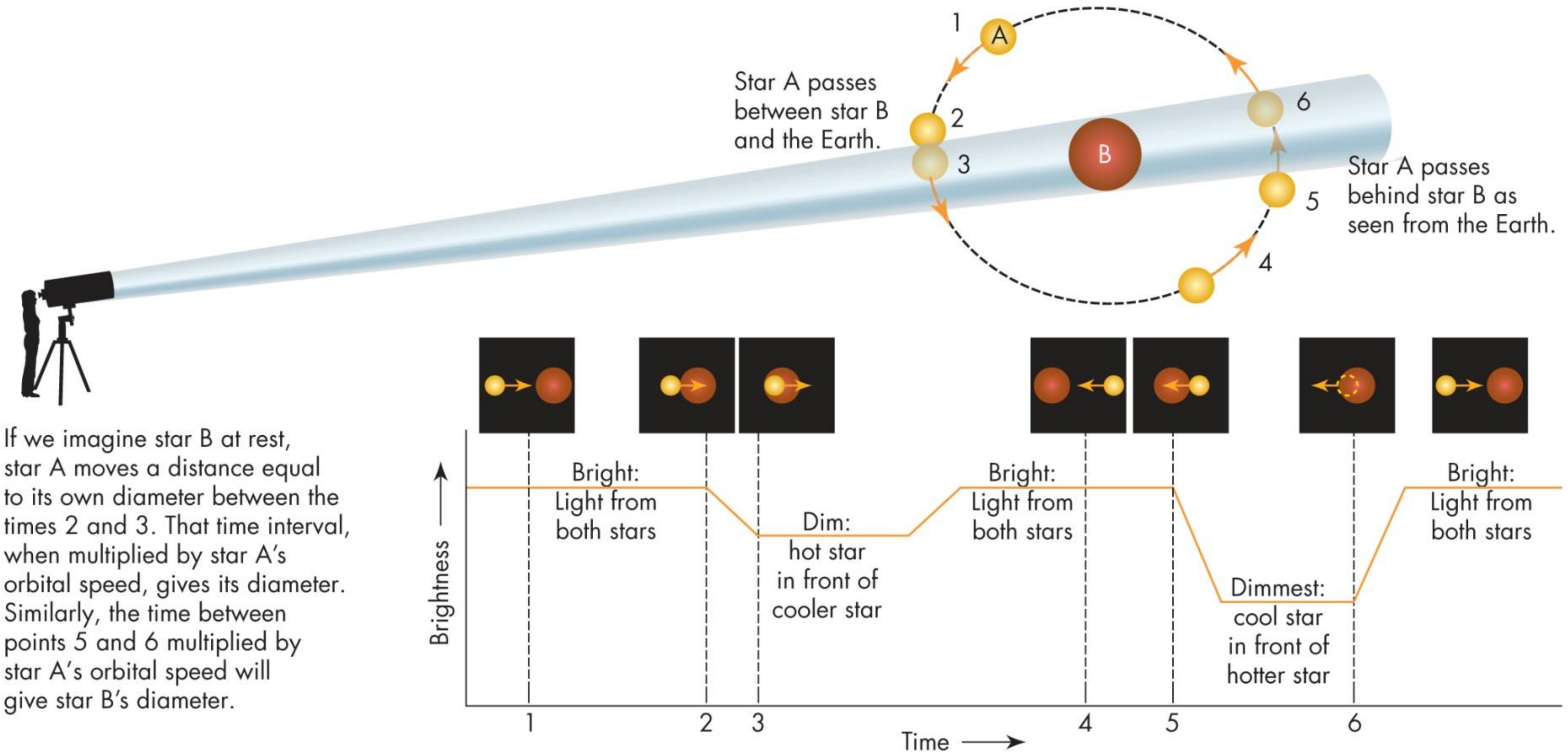
- P and a are determined from observations (may take a few years) and the above equation gives the combined mass (m + M).
- Further observations of the stars' orbit will allow the determination of each star's individual mass.
- Most stars have masses that fall in the narrow range 0.1 to 30 $M_\odot$.

Eclipsing Binaries



- A binary star system in which one star can eclipse the other star is called an ***eclipsing binary.***
- Watching such a system over time will reveal a combined light output that will periodically dim.

Stellar Transits



- The duration and manner in which the combined light curve changes together with the stars' orbital speed allows astronomers to determine the radii of the two eclipsing stars.

Summary of Stellar Properties

- Distance
  - Parallax (triangulation) for nearby stars (d less than 250 pc)
  - Standard-candle method for more distant stars
- Temperature
  - Wien's law (color-temperature relation)
  - Spectral class (O hot; M cool)
- Luminosity
  - Measure star's apparent brightness and distance and then calculate with inverse square law
  - Luminosity class of spectrum (to be discussed)
- Composition
  - Spectral lines observed in a star

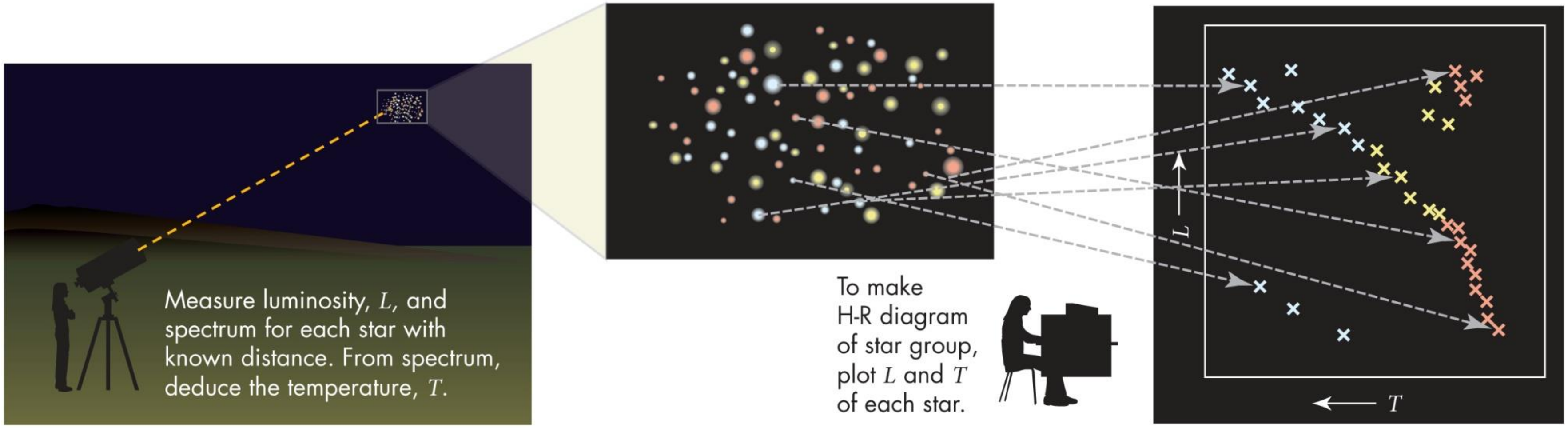
More Stellar Properties

- Radius
  - Stefan-Boltzmann law (measure L and T, solve for R)
  - Interferometer (gives angular size of star; from distance and angular size, calculate radius)
  - Eclipsing binary light curve (duration of eclipse phases)
- Mass
  - Modified form of Kepler's third law applied to binary stars
- Radial Velocity
  - Doppler shift of spectrum lines

Putting it all together – The Hertzsprung-Russell Diagram

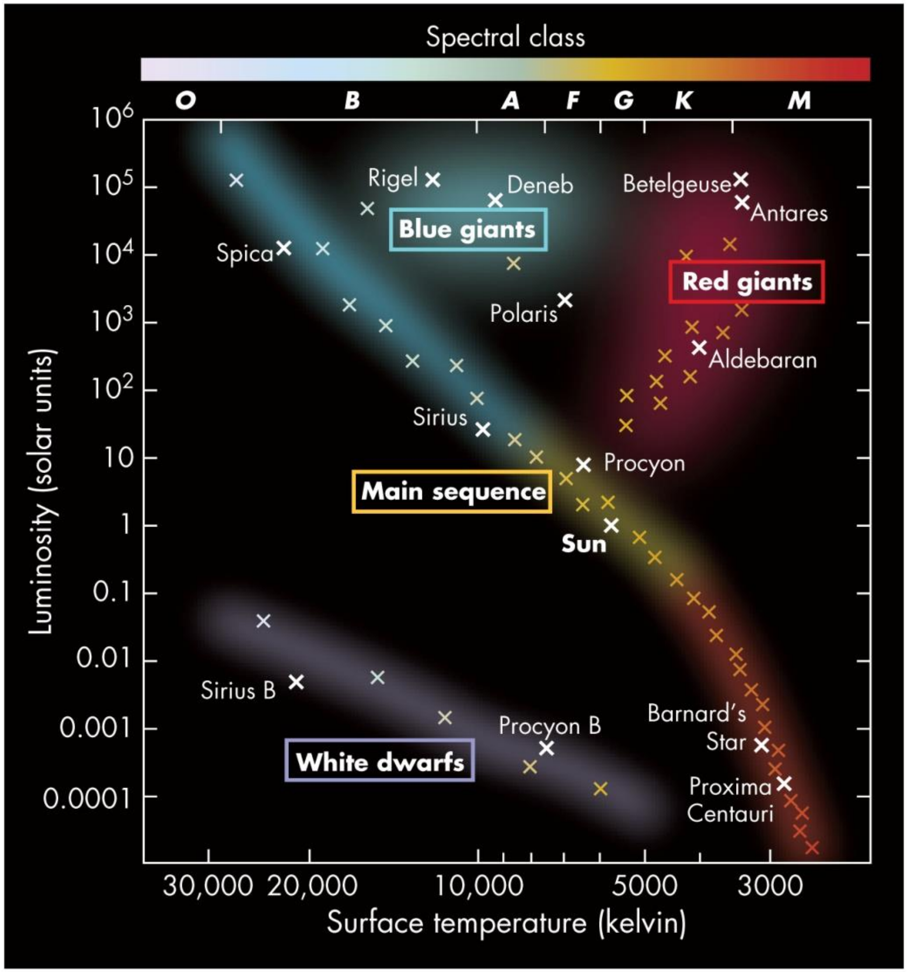
- So far, only properties of stars have been discussed – this follows the historical development of studying stars.
- The next step is to understand why stars have these properties in the combinations observed.
- This step in our understanding comes from the H-R diagram, developed independently by Ejnar Hertzsprung and Henry Norris Russell in 1912.

Constructing the HR Diagram

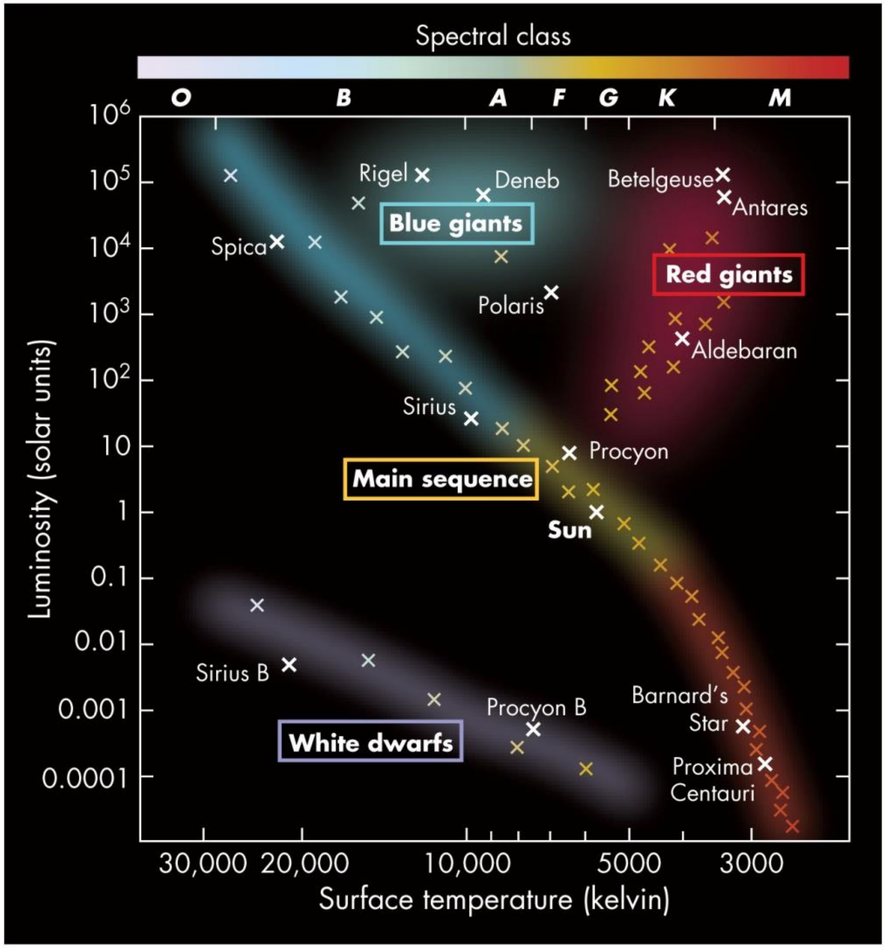


The HR Diagram

- The ***H-R diagram*** is a plot of stellar temperature vs. luminosity.
- Interestingly, most of the stars on the H-R diagram lie along a smooth diagonal running from hot, luminous stars (upper left part of diagram) to cool, dim ones (lower right part of diagram).

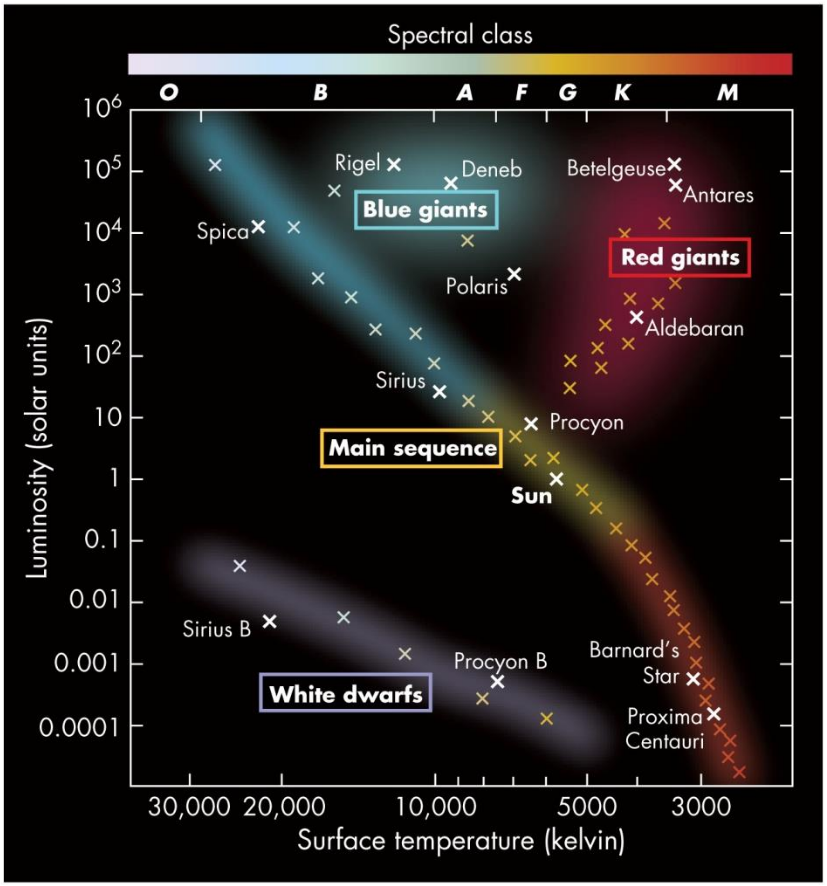


Axes of the HR Diagram



- By tradition, bright stars are placed at the top of the H-R diagram and dim ones at the bottom, while high-temperature (blue) stars are on the left with cool (red) stars on the right. (Note: temperature does not run in a traditional direction.)

The Main Sequence

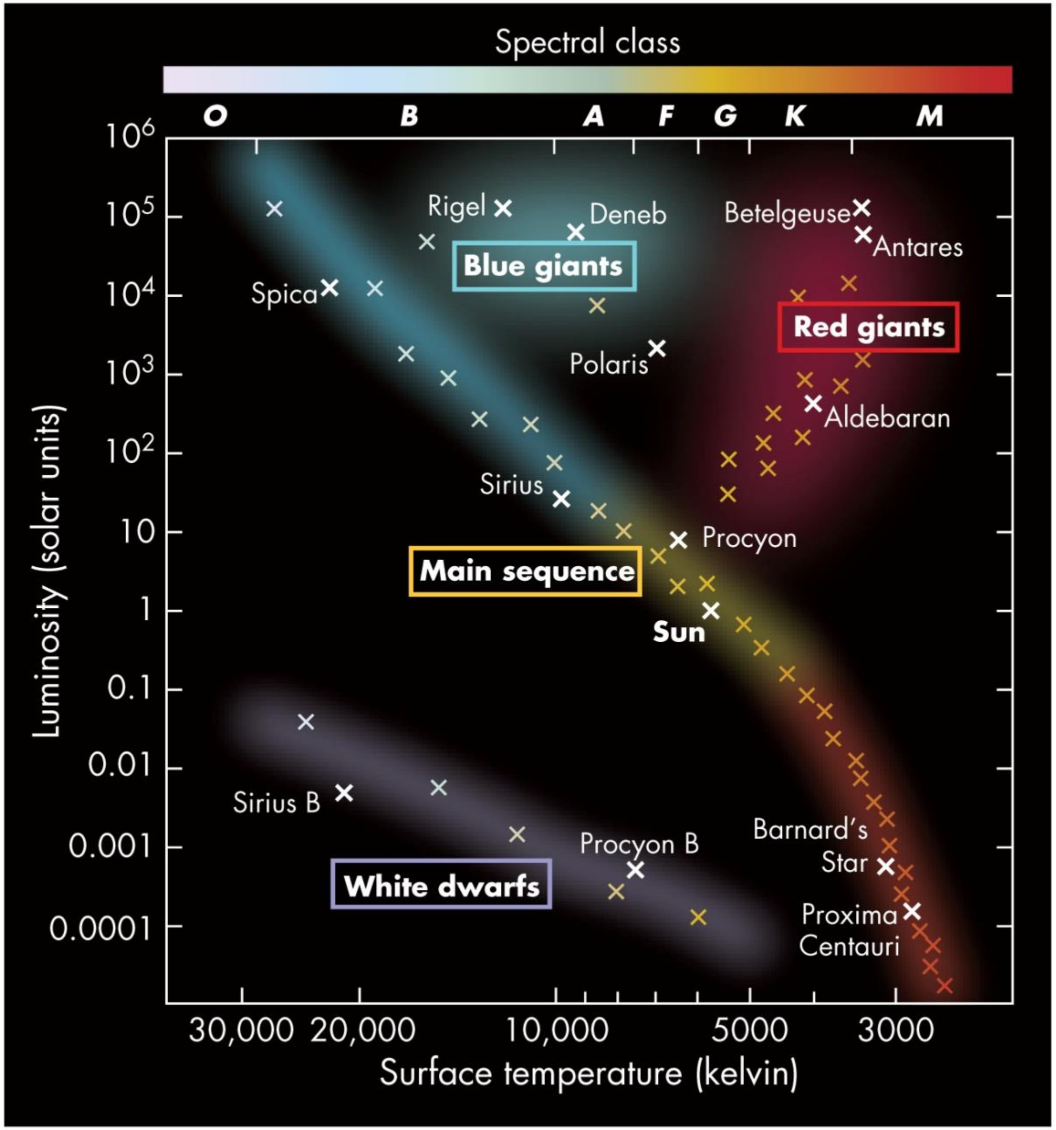


- The diagonally running group of stars on the H-R diagram is referred to as the ***main sequence.***
- Generally, 90% of a group of stars will be on the main sequence; however, a few stars will be cool but very luminous (upper right part of H-R diagram), while others will be hot and dim (lower left part of H-R diagram).

Understanding the HR Diagram

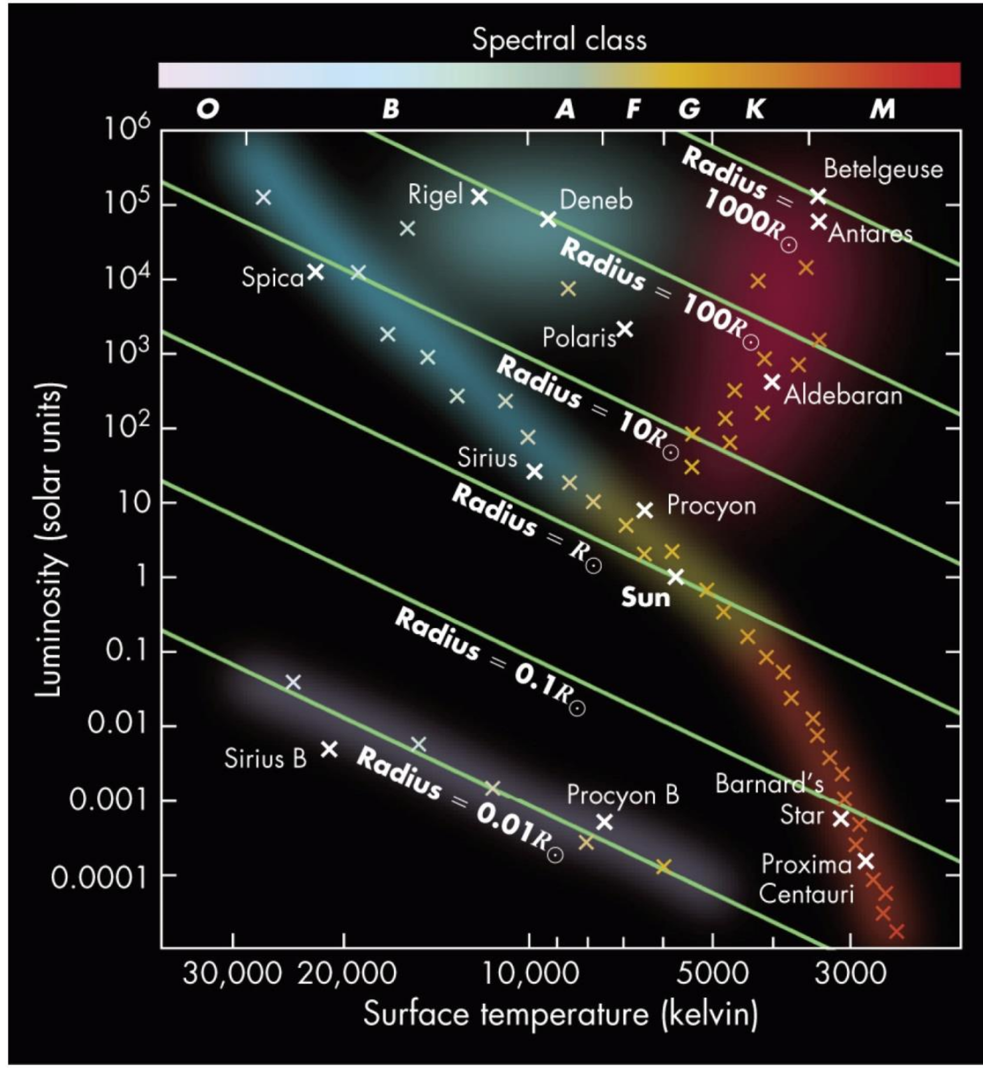
- The Stefan-Boltzmann law is a key to understanding the H-R diagram.
  - For stars of a given temperature, the larger the radius, the larger the luminosity.
  - Therefore, as one moves up the H-R diagram, a star’s radius must become bigger.
  - On the other hand, for a given luminosity, the larger the radius, the smaller the temperature.
  - Therefore, as one moves right on the H-R diagram, a star’s radius must increase.
  - The net effect of this is that the smallest stars must be in the lower left corner of the diagram and the largest stars in the upper right.

Interpreting the HR Diagram



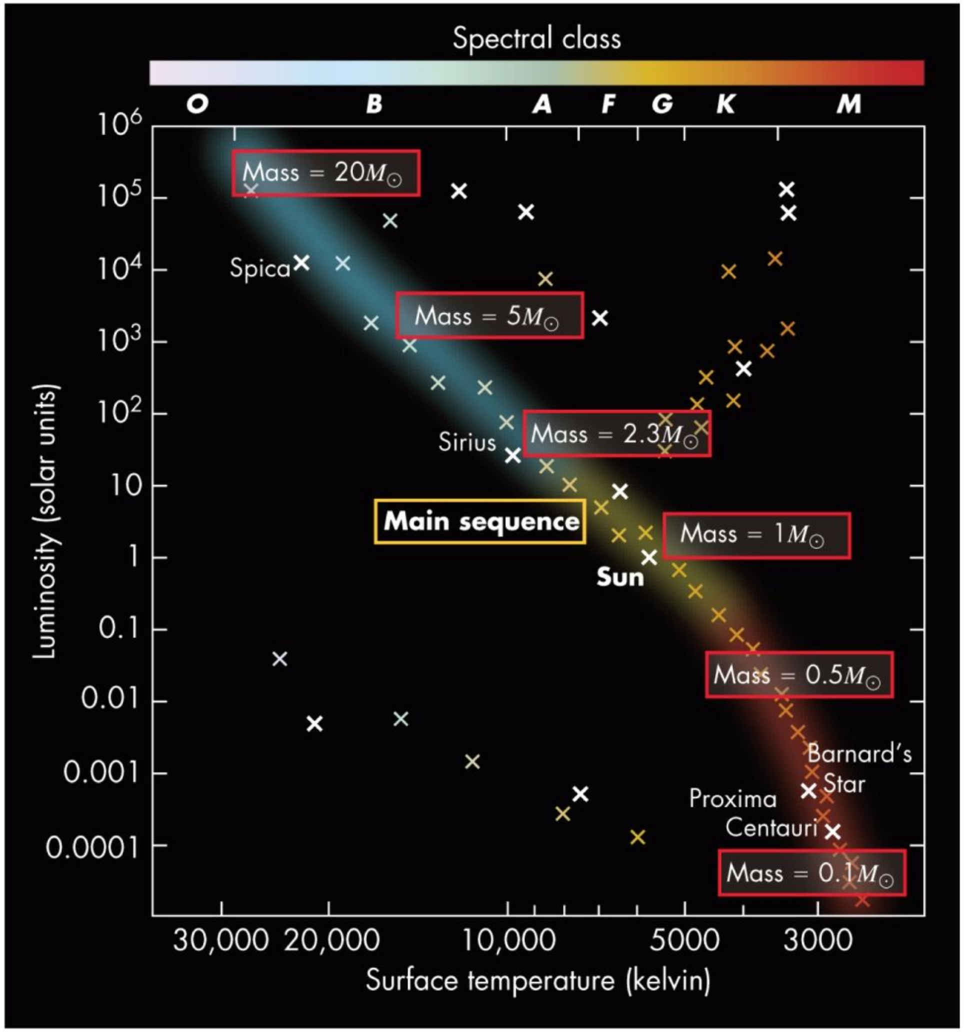
Giants and Dwarfs

- Stars in the upper left are called ***red giants*** (red because of the low temperatures there).
- Stars in the lower right are ***white dwarfs.***
- Three stellar types: main sequence, red giants, and white dwarfs

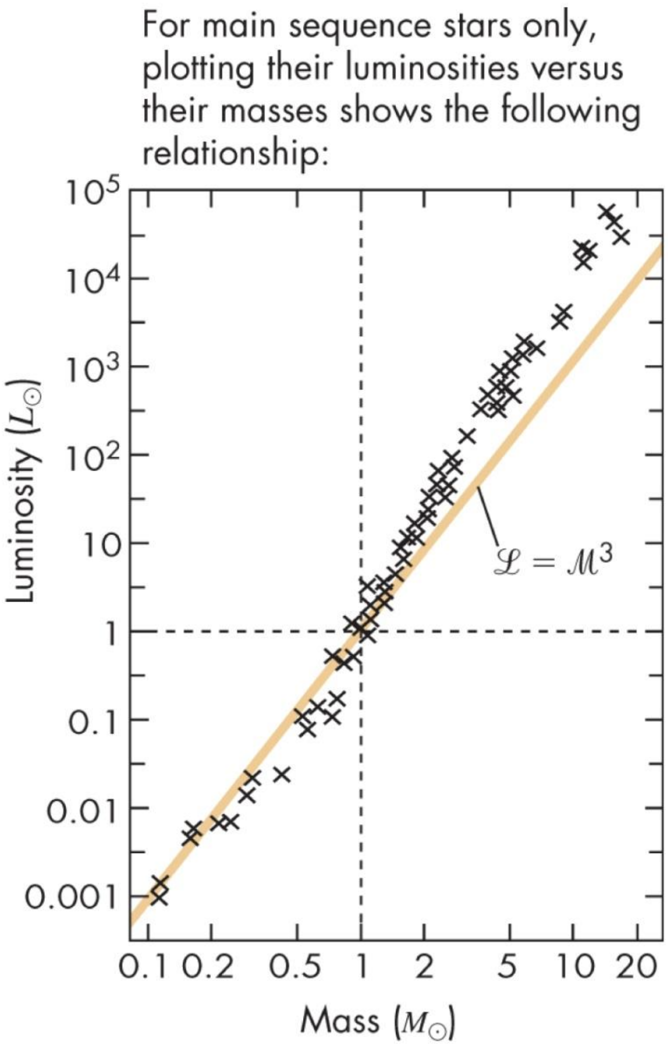


Mass and Density

- Giants, dwarfs, and main sequence stars also differ in average density, not just diameter.
- Typical density of main-sequence star is 1 g/cm^3, while for a giant it is 10^{-6} g/cm^3.



The Mass-Luminosity Relation



- Main-sequence stars obey a ***mass-luminosity relation,*** approximately given by:

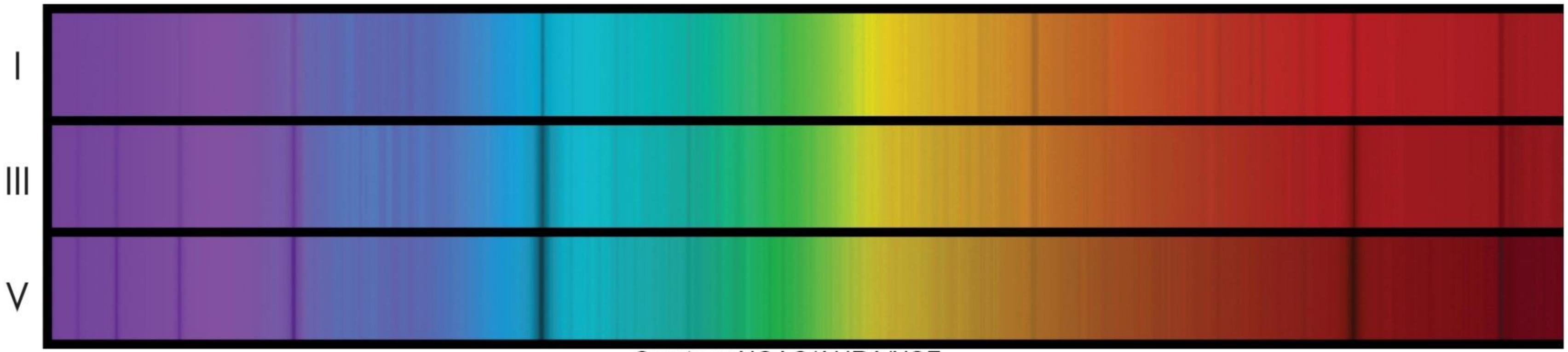
$$L = M^3$$

- *L* and *M* are measured in solar units
- Consequence: Stars at top of main-sequence are more massive than stars lower down.

Luminosity Classes

- Another method was discovered to measure the luminosity of a star (other than using a star's apparent magnitude and the inverse square law).
  - It was noticed that some stars had very narrow absorption lines compared to other stars of the same temperature.
  - It was also noticed that luminous stars had narrower lines than less luminous stars.
- Width of absorption line depends on density: wide for high density, narrow for low density.

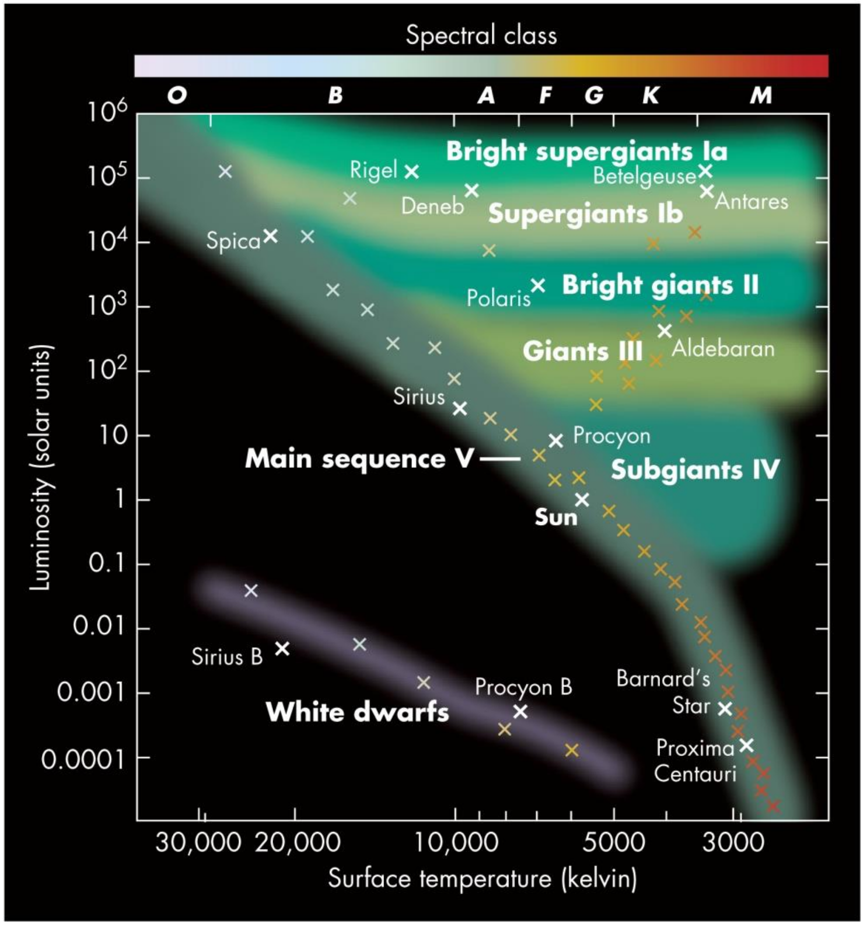
Using Spectral Lines



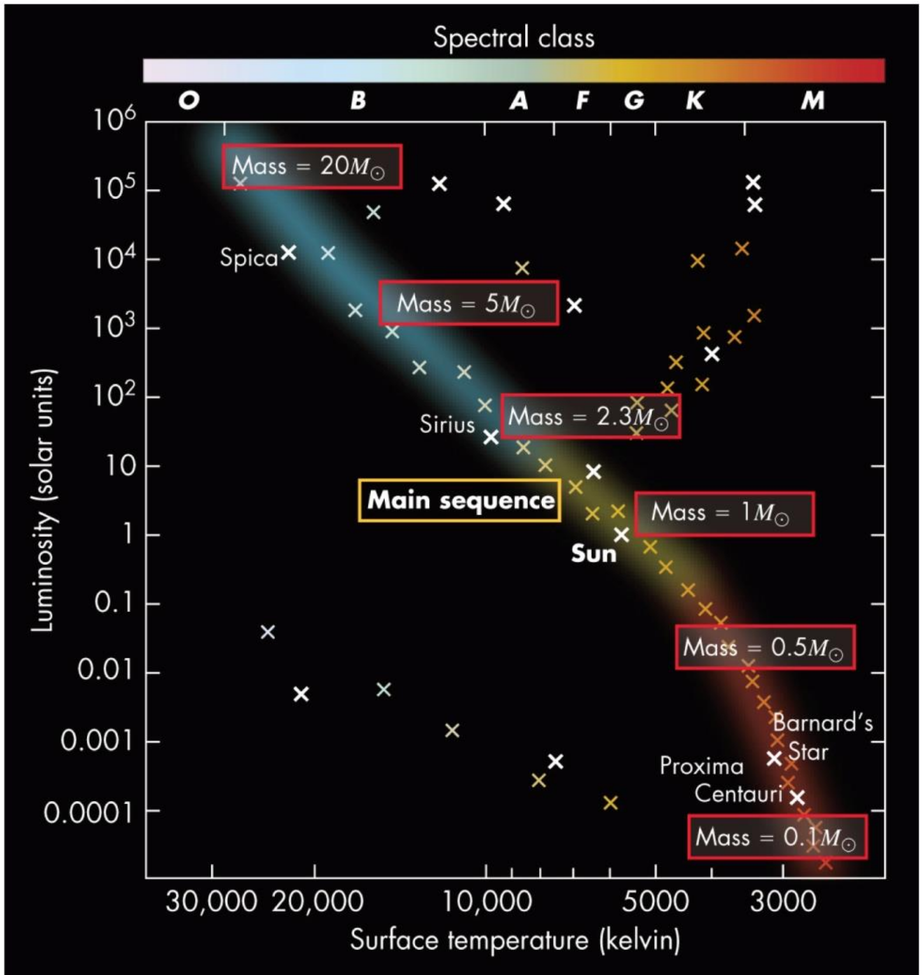
Courtesy NOAO/AURA/NSF

Classes of Stars on the HR Diagram

- Luminous stars (in upper right of H-R diagram) tend to be less dense, hence narrow absorption lines.
- H-R diagram broken into luminosity classes: Ia (bright supergiant), Ib (supergiants), II (bright giants), III (giants), IV (subgiants), V (main sequence)
  - Star classification example: The Sun is G2V

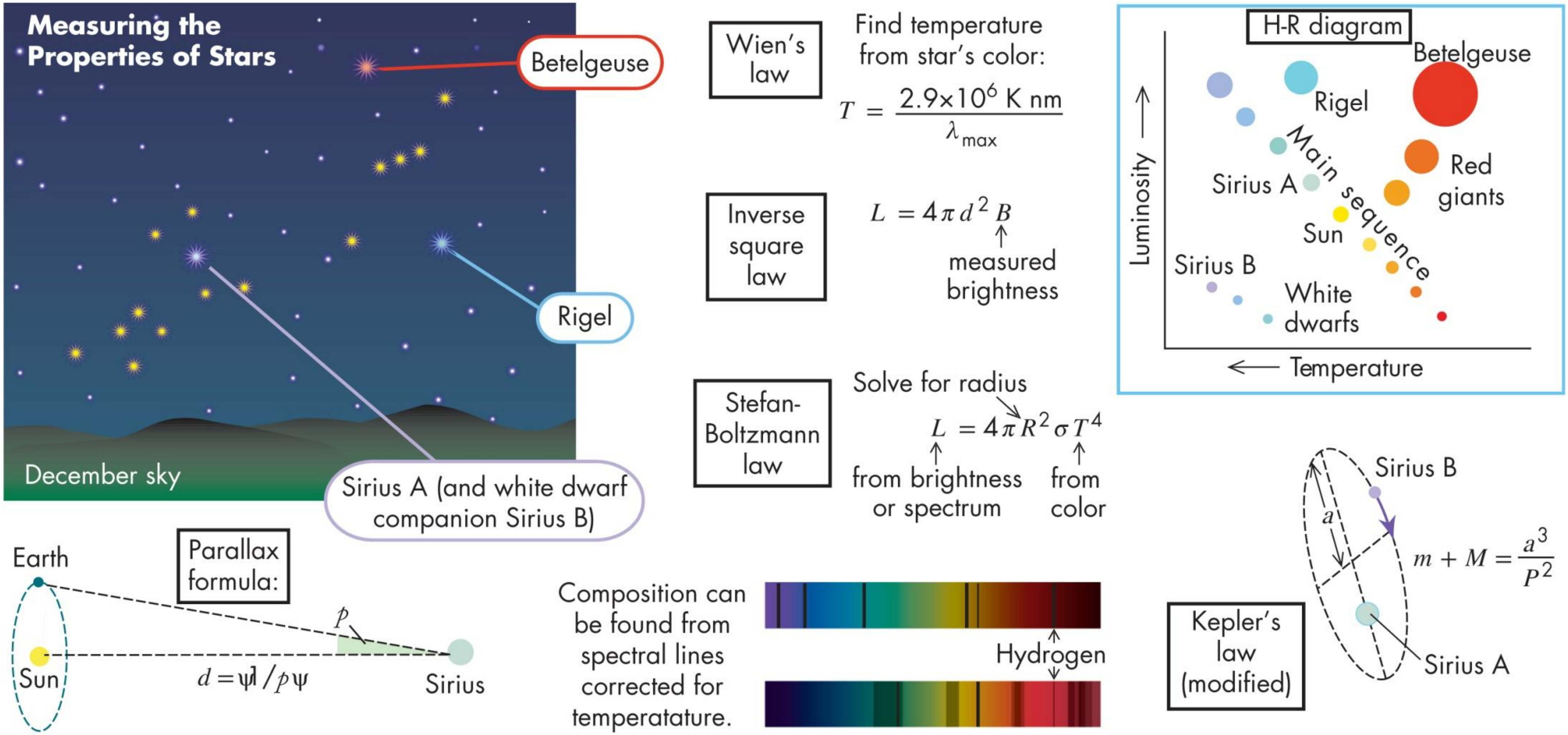


Summary of the HR Diagram



- Most stars lie on the main sequence
  - Of these, the hottest stars are blue and more luminous, while the coolest stars are red and dim
  - Star's position on sequence determines its mass, being more near the top of the sequence
- Three classes of stars:
  - Main-sequence
  - Giants
  - White dwarfs

Summary


Measuring the Properties of Stars
Betelgeuse
Rigel
December sky
Sirius A (and white dwarf companion Sirius B)
Wien's law
Find temperature from star's color:
$T = \frac{2.9\times10^6 \text{ K nm}}{\lambda_{max}}$
Inverse square law
$L = 4\pi d^2 B$
measured brightness
Stefan-Boltzmann law
Solve for radius
$L = 4\pi R^2 \sigma T^4$
from brightness or spectrum
from color
H-R diagram
Betelgeuse
Rigel
Main sequence
Red giants
Sirius A
Sun
Sirius B
White dwarfs
Luminosity
Temperature
Earth
Sun
Parallax formula:
p
d =ψ1/pψ
Sirius
Composition can be found from spectral lines corrected for temperatature.
Hydrogen
Kepler's law (modified)
Sirius B
a
$m + M = \frac{a^3}{P^2}$
Sirius A